PRACTICAL MATHEMATICS

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PART I

ARITHMETIC WITH APPLICATIONS

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PRACTICAL MATHEMATICS

BEING THE ESSENTIALS OF

ARITHMETIC, ALGEBRA
GEOMETRY AND TRIGONOMETRY

PART I
ARITHMETIC WITH APPLICATIONS

BY

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FOURTH EDITION

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III

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PREFACE TO THE FOURTH EDITION

This revision, for the most part, contains a new set of exercises. Some of the exercises that the late Dean Palmer selected with such great care have been changed but little, if at all. The data in the new problems have been brought up to date. A great many of them are entirely new in thought, and it is hoped that they may appeal to an even wider field of practical students. They conform to the present trend of practical applications of everyday problems in social science as well as technical science.

As in the earlier editions, all requests for aid will be answered, and all new exercises and criticism will be most cordially welcomed.

SAMUEL FLETCHER BIBB.

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PREFACE TO THE THIRD EDITION

This series of four books on practical mathematics was, perhaps, the pioneer set of its kind published in this country. The work grew during eight years in connection with the teaching of classes of practical men, where it was used in mimeographed form and rewritten several times previous to its first publication in 1912.

It was revised in 1918 and now appears in the third edition, in which a few topics have been omitted and many new ones added. The subject matter has been completely reset and much of it rewritten, the figures are all newly made, and the exercises and examples have been brought up to date with the addition of many new practical problems.

The series has been widely used in evening, trade, and continuation schools, and has found a place in the libraries of many practical men. The fact that the matter in these four books has been distributed to the number of more than 850,000 single volumes has encouraged the author and publishers in bringing out this third edition.

Usually when a man realizes for himself that mathematics is a powerful tool which he must be able to use in performing his work, he finds that even the arithmetic he learned at school has left him. It is for such that this series was written. The endeavor has been to make the student feel that he is in actual touch with real things, and at the same time to lay as broad a foundation as is consistent with the scope of the work.

Many of the problems are adapted from engineering and trade journals, from handbooks of various kinds, and from treatises on the steel square and other mechanical devices; and a large number of the especially practical ones are from the actual experience of members of the classes pursuing the course during its growth and of others who have corresponded with the author since the books were first printed.

This volume, which is Part I of the series, gives a concise treatment of various subjects in arithmetic and shows their applications. Methods for checking computations are given and various contracted processes are presented. In this edition a table of square roots is added, and the answers to the exercises are placed on the last pages of the book.

The author makes the rather unusual statement that he invites correspondence with students, especially those engaged in pursuits in which they apply mathematics. Experience shows that such correspondence is mutually profitable; the student may have his questions answered and the author often receives suggestions as to applications of mathematics. Constructive criticisms of the text or problems will be appreciated.

THE AUTHOR.

CHICAGO, ILL.,
March, 1930.

CONTENTS

PAGE

PREFACE TO THE FOURTH EDITION v

PREFACE TO THE THIRD EDITION vii

CHAPTER I

PRELIMINARY WORK AND REVIEW

ART.

1. Language of mathematics 1
2. Number 2
3. How to attack a problem 2
4. Definitions 3
5. Rules for finding divisors of numbers 4
6. Relative importance of signs of operation 7
7. Cancellation 10
8. Applying rules 14

CHAPTER II

COMMON FRACTIONS

10. Definitions 18
11. Mixed number 19
12. Proper and improper fractions 19
13. Comparison of fractions 19
15. Principles 21
16. Reduction of a whole or mixed number to an improper fraction 21
17. Reduction of an improper fraction to a whole or mixed number 22
18. Reduction of a fraction to lowest terms 22
19. Reduction of several fractions to fractions having the same denominator 23
20. Least common denominators 25
21. Addition of fractions 26
22. Subtraction of fractions 29

ART. PAGE
23. Multiplication of a fraction and an integer 32
24. Multiplication of a fraction by a fraction 33
25. Multiplication of mixed numbers 35
26. Division of a fraction by an integer 39
27. Division by a fraction 40
28. Special methods in division 41
29. Screws and screw threads 50
30. Pitch and lead of screw threads 50
31. Screw gearing 52

CHAPTER III

DECIMAL FRACTIONS

33. Historical note 57
34. Reading numbers 58
35. Reduction of a common fraction to a decimal fraction . 59
36. Decimal fraction to a common fraction 60
37. Addition of decimals 61
38. Subtraction of decimals 62
39. Multiplication of decimals 62
40. Division of decimals 63
41. Accuracy of results 63
42. Proportions of machine screw heads. A.S.M.E. standard 75
43. The micrometer caliper 78
44. The vernier 80
45. Micrometer with vernier 81

CHAPTER IV

SHORT METHODS AND CHECKS

46. Contracted methods and approximate results 84
47. Suggestive short methods 86
48. Multiplication 86
49. Division 88
50. Checking 89

CHAPTER V

WEIGHTS AND MEASURES

51. English system 93
52. Water pressure 98
53. The metric system 99

ART. PAGE
54. Measure of length. The meter 101
55. Legal units 101
56. Measure of surface 102
57. Measure of volume, cubic and capacity measures . . . 102
58. Measure of mass 102
59. Historical note 103
60. Terms used 103
61. Tables 104
62. Equivalents 106
63. Simplicity of the metric system 108
64. Relations of the units 109
65. Changing from English to metric or from metric to English system 110

CHAPTER VI

Percentage and Applications

66. Per cents as fractions 114
67. Cases 115
68. Rules and formulas 116
69. Solutions 116
70. Applications 119
71. Averages and per cent of error 120
72. List prices and discounts 121
73. Interest 128

CHAPTER VII

Ratio and Proportion

74. Ratio 131
75. Proportion 133
76. Principles and methods 134
77. Measuring heights 138
78. Steam-boiler proportions 141
79. The lever 142
80. Hydraulic machines 143

CHAPTER VIII

Density and Specific Gravity

81. Density 145
82. Specific gravity 145

ART. PAGE
83. Standards 145
84. Use 146

CHAPTER IX

POWERS AND ROOTS, SQUARE ROOT

85. Powers 150
86. Exponent of a power 150
87. Squares, cubes, involution 150
88. Roots 151
89. Radical sign and index of root 151
90. Square root 152
91. Process for the square root of a perfect square 152
92. Square root of a number containing a decimal 155
93. Approximate roots 156
94. Root of a common fraction 157
95. Short methods 157
96. Reasons for the methods 159
97. Rule for square root 160
98. Cube root 161
99. Use of table of square roots 162
100. Similar figures 164

Table I. Decimal and Fractional Equivalents of Parts of an Inch 167

Table II. English Inches into Millimeters 168

Table III. Specific Gravities and Weights of Substances 169

Table IV. Square Roots 171

Index 175

Answers 179

PRACTICAL MATHEMATICS

PART I

ARITHMETIC

CHAPTER I

PRELIMINARY WORK AND REVIEW

1. Language of mathematics.—Mathematics has a language of its own, with certain signs and symbols peculiar to it. It is as necessary to become familiar with these signs and symbols and their uses, in order to understand the language of mathematics, as it is for the shorthand writer to become familiar with the symbols used in his work. Failure to fix them in mind and to learn the definitions and technical terms keeps many students from mastering the mathematical subjects they take up.

The language of mathematics is one of signs and symbols and, in a sense, is an unspoken language. There can be no more universal or more simple language, no language more exempt from error and obscurity. The language of mathematics is the same throughout the civilized world, though the people of each country translate it into their peculiar spoken language.

Some of the best known symbols of mathematics are the Arabic numerals, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, the signs of addition, $+$, subtraction, $-$, multiplication, $\times$.

division, $\div$, and equality, $=$. Other symbols will be explained as used.

In the study of mathematics much time should be devoted (*a*) to the expressing of verbally stated facts in mathematical language, that is, in the signs and symbols of mathematics, and (*b*) to the translating of mathematical expressions into common language.

The signs and symbols of mathematics are used for convenience. They have gradually come into use by general agreement. In some cases the symbols are abbreviations of words, but often have no such relation to the thing they stand for. We cannot tell why they stand for what they do any more than we can tell why the words for cat and dog stand for the different animals they do. They mean what they do by **common agreement** or by **definition.**

2. Number.—Arithmetic deals with the relations and combinations of numbers and deals with any or all practical problems to which it may be applied by means of numbers. What then is a number? This question is not easily answered. Perhaps the best we can do is to urge that each person should endeavor to make clear to himself the meaning of number and whence came the numbers that are connected with a particular problem. A number may come from counting the individuals in a group of objects, or a number may be used in stating the amount or measure of something that has been measured. Thus, one measures the length of a room and finds that it is 30 feet.

3. How to attack a problem.—A problem in mathematics should not be attacked as a puzzle. No guesswork has any place in its consideration. The statement of the problem should be clear and so leave but one

solution possible. To state the problem clearly is the duty of the author or the one who proposes the problem. The following suggestions should be carefully observed by the student:

(1) *The problem should be read and analyzed so carefully that all conditions are well fixed in mind.* If the problem cannot be understood there is no use in trying to solve it. Of course, if the answer is given, a series of guess-operations may obtain it, but the work is worse than useless.

(2) *In the solution there should be no unnecessary work.* Shorten the processes whenever possible.

(3) *Always apply some proof or check to the work if possible.* A wrong answer is valueless. Accuracy is of the highest importance, and to no one more than to the practical man. If a check can be applied there is no need of an answer being given to the problem.

4. Definitions.—In order to be exact in ideas and statements, it is necessary to give certain definitions. It would seem, however, that for the practical man, technical terms should be omitted so far as possible. It is usually sufficient to make the term understood, though the definition may not be a good one technically. In mathematics more than in almost any other subject, each word used has a **definite** and **fixed meaning.**

The following definitions are inserted here to help recall to mind some of the terms used:

(1) An **integer** or an **integral number** is a whole number.

(2) A **factor** or a **divisor** of a whole number is any whole number that will exactly divide it.

(3) An **even number** is a number that is exactly divisible by 2.

Thus, 4, 8, and 20 are even numbers.

(4) An **odd number** is an integer that is not exactly divisible by 2.

Thus, 5, 11, and 47 are odd numbers.

(5) A **prime number** is a number that has no factors except itself and 1.

Thus, 1, 2, 7, 11, and 17 are prime numbers.

(6) A **composite number** is a number that has other factors than itself and 1.

Thus, 6, 22, 49, and 100 are composite numbers.

(7) A **common factor** or **divisor** of two or more numbers is a factor that will exactly divide each of them. If this factor is the largest factor possible, it is called the **greatest common divisor**: abbreviated to G. C. D.

Thus, 4 is a common divisor of 16 and 24, but 8 is the G. C. D. of 16 and 24.

(8) A **multiple** of a number is a number that is divisible by the given number. If the same number is exactly divisible by two or more numbers, it is a **common multiple** of them. The least such number is called the **least common multiple** and is abbreviated to L. C. M.

Thus, 36 and 72 are common multiples of 12, 9, and 4, but 36 is the L. C. M.

5. Rules for finding divisors of numbers.—It is often convenient to be able to tell without performing the division, whether or not a given number is divisible

by another. The following rules will assist in this. Their proofs are not given here.

(1) A number is divisible by 2 if its right-hand figure is 0 or one divisible by 2.

(2) A number is divisible by 3 if the sum of its digits is divisible by 3.

Thus, 73,245 is divisible by 3 since $7+3+2+4+5=21$ is divisible by 3.

(3) A number is divisible by 4 if the number represented by its last two digits on the right is divisible by 4, or if it ends in two zeros.

Thus, 87,656 is divisible by 4 since 56 is divisible by 4.

(4) A number is divisible by 5 if the last figure on the right is 0 or 5.

(5) An even number the sum of whose digits is divisible by 3 is divisible by 6.

(6) No convenient rule can be given for 7; the best thing to do is to test by trial.

(7) A number is divisible by 8 if the number represented by the last three digits on the right is divisible by 8.

Thus, 987,672 is divisible by 8 since 672 is divisible by 8.

(8) A number is divisible by 9 if the sum of its digits is divisible by 9.

(9) A number is divisible by 11 if the difference between the sums of the odd digits and the even digits, counting from the right, is divisible by 11.

Thus, 47,679,291 is divisible by 11 since $(9+9+6+4)-(1+2+7+7)=11$ is divisible by 11.

This rule is of little value since the division can be tried about as easily as the rule can be applied.

The following facts are of some value:

(10) A factor of a number is a factor of any of its multiples.

(11) A common factor of any two numbers is a factor of the sum or the difference of any two multiples of the numbers.

Remark.—The 11 statements above are not meant to be all-inclusive. They are principles that are found to be the most practical when *integral numbers* are being used in arithmetic.

EXERCISES

1. Name the even numbers and the odd numbers in the following: 4, 11, 20, 29, 555, 1166, 1939, 1940.

2. Which of the following are prime numbers and which are composite: 7, 11, 18, 21, 23, 46, 69, 71, 1492, 1777, 1939?

3. Some of the prime numbers are given (*a*) by $2+1$, $4+1$, $16+1$, $256+1$, etc. (2^n+1, n a power of 2); (*b*) by $2-1$, $8-1$, $32-1$, $128-1$, etc. (2^p-1, p itself a prime).

4. Give all the divisors besides 1 and itself of 6; of 24; of 120; of 720; of 5040; of 10; of 70; of 770.

5. Is 39,916,800 divisible by 2? by 3? by 4? by 5? by 6? by 7? by 8? by 9? by 10? by 11?

Find the common divisors and the greatest common divisor of each of the following groups of numbers:

6. 12, 16, 24.

7. 16, 24, 32.

8. 9, 27, 144.

9. 110, 220, 330.

10. 24, 720, 5040.

11. 65, 780, 1190.

Find the least common multiple of each of the following groups:

12. 2, 4, 6, 8, 10.

13. 1, 3, 5, 7, 9.

14. 3, 11, 99, 726.

15. 3, 7, 63, 147.

16. 5, 15, 75, 225.

17. 7, 22, 186, 3388.

18. Find the prime factors of each of the following: 756, 2376, 7920, 65,520.

19. Tell which of the following numbers are exactly divisible by each of 2, 3, 4, 5, 6, 7, 8, and 9: 91,008; 6,370,560; 12,731,040; 181,872; 32,670; 22,869,000.

20. Give three divisors of 252. Give three multiples of 75. Give three common multiples of 21, 28, 9. Give the least common multiple of the last three numbers. Is there a greatest common multiple of the last three numbers?

21. Is 360 multiplied by any integer n divisible by each of 2, 3, 4, 5, 6, 8, and 9?

22. Is 360 multiplied by $(a+b)$ divisible by each of 2, 3, 4, 5, 6, 8, and 9? Multiplied by $(a-b)$?

6. Relative importance of signs of operation.

(1) A series of additions may be taken in any order.

Thus, $4+5+7+3=19$; when added in any order.

(2) A series of subtractions *must* be taken as written.

Thus, $100-20-10-3=67$, when taken in the order written.

(3) A series of multiplications may be taken in any order.

Thus, $2\times3\times5\times6=180$, when taken in any order.

(4) A series of divisions *must* be taken as written.

Thus, $100\div10\div2=5$, when taken in the order written.

(5) In a series of different operations, the multiplications *must* be performed first, the divisions next, and then the additions and subtractions may be taken in the order written.

Examples.—(1) $12+3-2+9+7-3=26$, by performing the operations in the order in which they occur.

(2) $120\div3\times5\times2\div2=2$, by first performing the multiplications and then the divisions in the order in which they occur.

(3)
$$\begin{aligned} &12\div3+8\times2-6\div2+7\times2\times3-9\\ &=12\div3+16-6\div2+42-9\\ &=4+16-3+42-9=50 \end{aligned}$$

by first performing the multiplications, then the divisions, and then the additions and subtractions.

Remark 1.—The above methods of operations will give the correct result but in a series of subtractions the numbers that are to be subtracted could be subtracted one at a time in any order.

Thus, $100-20-10-3=100-3-20-10=100-3-10-20=67$.

Or all of the numbers that are to be subtracted could be added to form a single sum (number) and then this single number could be subtracted.

Thus, $100-(20+10+3)=100-33=67$.

Remark 2.—In a series of different operations the parentheses ( ) and brackets [ ] could be used to group the desired order of operations.

Thus, $120\div3\times5\times2\div2=[120\div(3\times5\times2)]\div2=[120\div30]\div2=4\div2=2$.

NOTE.—The operation indicated by the shortest group is performed first.

EXERCISES

In Exercises 1 to 8 remember that the operations indicated by the shortest group should be performed first.

1. $(18+19-8)+(12-8-4)=?$

2. $[21\div(7\times3)]+(4\times5)=?$

3. $18-[(3+6+9)\div(9-6)]-12=?$

4. $[(25-4-6)\div(3\times5)]+4\times3=?$

5. $(27-7)-[(2\times3)\div(3\times2)]-3\times6=?$

6. $[(55-10)\times(3\times6\times90)]\div[(4\times10)+5]=?$

7. $[256\div(16\times16)]+[225\div(15\times1)]=?$

8. $(720\div360)+(180\times2)+1=?$

9. Time yourself in doing the following six multiplications:

(1) 135×246
(2) 157×268
(3) 790×281
(4) 975×864
(5) 793×682
(6) 9753×6429

10. Check your work in the above six multiplications by doing the multiplying, using the first number in each case as the multiplier.

11. Time yourself in doing the following divisions. Check your work by finding the product of the divisor and quotient and comparing it with the dividend.

(1) $549{,}198\div9$.
(2) $427{,}154\div7$.
(3) $5746\div26$.
(4) $29{,}375\div235$.
(5) $32{,}277\div371$.
(6) $876{,}543{,}210\div90$.

12. Do the following multiplications and check the work by dividing the product by the multiplier and comparing the result with the multiplicand:

(1) 384×923.
(2) 2374×879.
(3) 4943×4350.
(4) 8793×7329.

13. Do the following divisions and check the work by finding the product of the divisor and quotient, then adding the remainder and comparing the result with the dividend:

(1) $93{,}462\div79$.
(2) $637{,}842\div2327$.
(3) $647{,}847\div353$.
(4) $9{,}963{,}486\div573$.

14. Note the following interesting products:
Multiply 12,345,679 by

(1) 63, which is 7×9.
(2) 54, which is 6×9.
(3) 45, which is 5×9.
(4) 36, which is 4×9.

Write down the results for 27, 18, and 9.

15. If 650 airships cost \$48,002,500, what is the cost per ship?

16. For 1938 the United States produced 5,008,000 fine ounces of gold valued at \$175,280,000. What is the value of 1 fine ounce?

17. There are 21 accidents in the United States per minute. How many accidents will there be in the United States in 365 days?

18. In order to do satisfactory work, the teeth of a saw must travel about 9225 feet per minute. A circular saw has 75 teeth 1 inch apart. How many revolutions a minute should the saw make?

19. The circumference of a drive wheel of a locomotive is 22 feet. How many revolutions will it make in going 286 miles if there are 5280 feet in a mile?

20. The pressure at the center of the earth is estimated to be about 3,200,000 atmospheres. If this is 32 times the greatest pressure ever produced in a laboratory, and if the atmospheric pressure is 15 pounds per square inch, find the greatest pressure ever produced in a laboratory.

21. If a man takes a step of 28 inches and can walk 7 miles in 2 hours, how many steps does he take per minute (5280 feet in a mile)?

PROBLEMS INVOLVING MULTIPLICATION AND DIVISION

7. Cancellation.—Often in solving problems, a fractional form like the following is obtained:

$$\frac{64\times25\times8\times12\times17}{48\times15\times32\times17\times24}$$

If we do all the multiplications above the line and below the line, and then perform the division which the line indicates, we shall obtain the result. It is often easy to avoid much of this work, however, by applying a principle of fractions. The process which is explained below is called **cancellation:**

$$\frac{\overset{2}{\cancel{64}}\times\overset{5}{\cancel{25}}\times\cancel{8}\times\cancel{12}\times\cancel{17}}{\underset{6}{\cancel{48}}\times\underset{3}{\cancel{15}}\times\cancel{32}\times\cancel{17}\times\underset{2}{\cancel{24}}}$$

(1) It is seen that 17 is found both above and below the line; we draw a line through each of these. These numbers are then said to be canceled.

(2) Now notice that the numbers 64 and 32 are divisible by 32. Cancel 64 and 32 and place 2 above 64 which is the number of times 32 is contained in 64.

(3) Next, divide 48 and 8 by 8 and cancel them, writing the quotient 6 below 48.

(4) Divide 25 and 15 by 5 and cancel them, writing the quotient 5 above 25 and 3 below 15.

(5) In a similar manner 12 and 24 are canceled; also 2 and 2.

In this manner we have replaced the given form by the simpler one $\frac{5}{6\times3}=\frac{5}{18}$. This is the answer.

It should be noted that when no factor remains either above or below the line after the cancellation is finished, we retain one of the unit factors which we neglected to write when canceling.

Thus, $\frac{\cancel{4}\times\cancel{8}\times\cancel{16}\times\cancel{10}}{\underset{3}{\cancel{12}}\times\underset{10}{\cancel{80}}\times\underset{3}{\cancel{48}}}=\frac{1}{9}$.

Remark.—It should be remembered that this method of simplifying cannot be used when there are additions or subtractions indicated in the problem.

In such a case the operations above the line must be performed first, then those below, and finally the result above must be divided by the result below.

Thus, $\frac{4+200-6\times2}{38+98-12\times6}=\frac{192}{64}=3.$

These processes may be restated in the following:

RULE.—(1) *Any factor above may be divided into any factor below the line.*

(2) *Any factor below may be divided into any factor above the line.*

(3) *Any factor common to factors one above and one below may be divided into each.*

(4) *The answer is obtained by dividing the product of the numbers remaining above the line by the product of the numbers remaining below the line. If no number remains above or below, use* 1.

EXERCISES

Use cancellation to find the results in the following:

1. $\frac{3\times9\times7\times18\times5}{9\times21\times15}$.
2. $\frac{30\times56\times24}{12\times14\times10}$.
3. $\frac{68\times121\times38}{17\times22\times19}$.
4. $\frac{88\times200\times16\times28}{12\times11\times10\times9\times8}$.
5. $\frac{36\times50\times26\times24}{15\times39\times10}$.
6. $\frac{12\times14\times16\times51}{34\times64\times28}$.
7. $\frac{99\times888\times77}{72\times777\times42}$.
8. $\frac{2100\times155\times60\times100}{700\times31\times28}$.
9. $\frac{120\times135\times510\times57}{36\times33\times170\times30}$.
10. $\frac{570\times5400\times6250}{45\times60\times60\times90}$.

11. $\dfrac{288\times800\times5280}{48\times540\times5600}$.

12. $\dfrac{1020\times234\times1212-1020\times101}{[180+(3\times90)+570]24\times101}$.

13. $\dfrac{5\times15+3\times25}{3\times50}$.

14. $\dfrac{(77-15)+2\times19}{(2\times14)-(3\times8)}$.

15. $\dfrac{(17\times33)-(17\times11)+(17\times44)}{(17\times11)[66-(4\times15)]}$.

16. $\dfrac{512\times6+[256\times6-(32\times12)]}{(6\times64)[(256\times6)-(16\times24)]}$.

Analyze the following and shorten the computation as much as possible by cancellation.

17. If 18 men can do a piece of work in 14 days, how many men will do the work in 21 days?

Operation

$$\frac{\overset{6}{\cancel{18}}\times\overset{2}{\cancel{14}}}{\underset{3}{\cancel{21}}}=12$$

Analysis.—If 18 men can do a piece of work in 14 days, one man can do the work in 18×14 days. It will take as many men to do the work in 21 days as 21 is contained times in 18×14.

Hence, it takes 12 men to do the work in 21 days.

18. If 48 electric fans cost $312, what will 18 of these fans cost?

19. If a man trades 27 dozen eggs valued at 12 cents per dozen for bacon valued at 18 cents per pound, how many pounds of bacon does he get?

20. A man walks 28 miles in 8 hours. If he takes steps of 28 inches, how many steps per minute does he take?

21. If 48 screws weigh 1 pound, how many boxes each containing 51 screws can be filled from 68 pounds of screws?

22. If six men drink 124 quarts of milk in 31 days, how many quarts will eight men drink in 24 days?

23. A man drives 256 miles on gasoline that costs 18 cents per gallon and finds that he is getting 16 miles to the gallon. How many miles could he have traveled on a train at 3 cents per mile if the cost is the same as for the car trip?

24. A farmer trades corn valued at 72 cents a bushel for a tank of gasoline that holds 128 cubic feet. If the gasoline is valued at 18 cents per gallon, and if there are $7\frac{1}{2}$ gallons to the cubic foot, how many barrels of corn were traded if there are 5 bushels in a barrel?

25. If 24 men in 18 days of 8 hours each can dig a ditch 105 rods long, 12 feet wide, and 9 feet deep, how many men in 24 days of 12 hours each will be required to dig a ditch 420 rods long, 9 feet wide, and 6 feet deep?

26. A bookstore orders 36 books valued at \$1.50 per book and 48 books valued at \$2.10 per book. How many different books valued at \$3.60 per book must the store return to pay for the books ordered?

27. A dealer orders 24 coats valued at \$5.46 per coat and 36 pairs of shoes valued at \$4.16 per pair. To pay for the coat and shoes, the dealer returns enough golf balls valued at 52 cents per ball. How many dozen golf balls were returned?

28. A room 30 feet long, 30 feet wide, and 10 feet high contains about 700 pounds of air. What is the weight of air in another room under conditions similar to the first if it is 120 feet long, 90 feet wide, and 12 feet high?

8. Applying rules.—The practical man often has to apply a rule in solving a problem. This rule may be given to him by a fellow workman, or it may be taken from a handbook. The rule may be one, the reasonableness of which is apparent, but often it is not. Many

rules are the results of experience, others of experiment, and still others are mere "rules of thumb," that is, they merely state a combination of numbers which gives the result desired.

In the following problems, read the rule carefully and be sure it is understood before applying it.

EXERCISES

RULE.—To find the number of revolutions of a *driven* pulley in a *given* time, multiply the *diameter* of the *driving* pulley by its number of *revolutions* in the given time and divide by the *diameter* of the *driven* pulley.

1. A pulley 45 inches in diameter and making 68 revolutions per minute (R. P. M.) is driving a pulley 51 inches in diameter. Find the number of R. P. M.

2. Find the R. P. M. of a pulley 9 inches in diameter driven by a 24-inch pulley making 48 R. P. M.

3. Find the R. P. M. of a pulley 52 inches in diameter driven by a 36-inch pulley making 78 R. P. M.

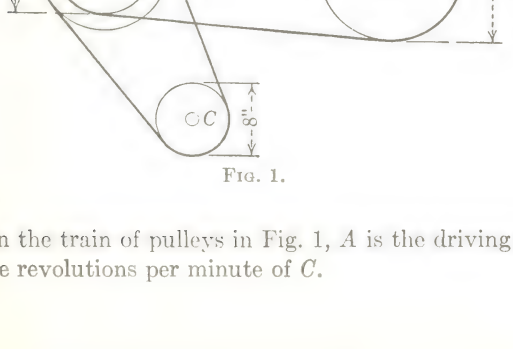

FIG. 1.

4. In the train of pulleys in Fig. 1, A is the driving pulley. Find the revolutions per minute of C.

5. In the pulley train of Fig. 2, find the revolutions per minute of the lathe pulley.

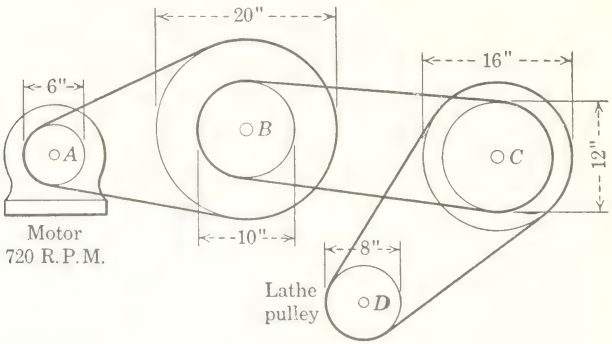

FIG. 2.

RULE.—To determine the width of belt required to transmit a given **horsepower** at a given speed of the belt: For **single leather** or **four-ply rubber belts,** multiply the number of horsepower to be transmitted by 33,000 and divide the product by the product of the speed of the belt, in feet per minute, multiplied by 60. The quotient will be width of the belt in inches.

6. What is the required width of belt to transmit 100 horsepower with a belt speed of 3500 feet per minute?

Solution

$$\frac{\overset{110}{\cancel{100}}\times\overset{550}{\cancel{33000}}}{\underset{7}{\cancel{3500}}\times\cancel{60}} = \frac{110}{7} = 15\tfrac{5}{7}\text{in.}$$

7. Find the width of a single leather belt to transmit 84 horsepower, with a belt speed of 4400 feet per minute.

8. For heavy double leather of six-ply rubber belts, use 100 instead of 60 in the rule. Find the width of such a belt to transmit 120 horsepower with a belt speed of 3200 feet per minute.

RULE.—To determine the horsepower that a belt of given width will transmit when running at a given speed: For **single leather** or **four-ply rubber belts,** multiply width of belt in inches by 60 and the product by speed of belt in feet per minute and divide the product by 33,000. The quotient will be the number of horsepower that the belt will transmit with safety.

9. How many horsepower will an 8-inch single leather belt transmit if running at 4400 feet per minute?

10. How many horsepower will a 32-inch heavy double leather belt transmit if running at 4000 feet per minute? (Use 100 instead of 60 in the rule.)

The first letter of a word is often used in mathematics instead of the word itself. When two or more such letters are written together with no sign between them, it is understood that multiplication is indicated.

If

H = horsepower.

P = effective pressure in pounds of steam per square inch.

L = length of piston stroke in feet.

A = area of piston in square inches.

N = number of strokes per minute.

Then the **rule** for finding the horsepower of a steam engine may be stated in the following abbreviated form:

$$H = \frac{PLAN}{33000}$$

11. Find H if $P = 55$ pounds per square inch, $L = 2$ feet, $A = 195$ square inches, and $N = 80$.

Solution

$$H = \frac{55 \times 2 \times 195 \times 80}{33000} = 52$$

12. Find H if $P = 66$, $L = 2$, $A = 180$, and $N = 100$.

13. Find H if $P = 99$, $L = 2$, $A = 85$, and $N = 190$.

CHAPTER II

COMMON FRACTIONS

DEFINITIONS AND GENERAL PROPERTIES

9. The number 6 when divided by 3 gives a quotient of 2. This may be written $\frac{6}{3}=2$. If now we attempt to divide 6 by 7, we are unable to find the quotient as above. The division may be written $\frac{6}{7}$. This is called a **fraction.** The fraction $\frac{6}{7}$ represents a number, but it is not a whole number. Our idea of number must now be enlarged to include fractions.

$\frac{1}{7}$ means that a unit is divided into 7 equal parts. The fraction $\frac{6}{7}$ indicates that 6 of the 7 equal parts are taken.

10. Definitions.—A **fraction** is an indicated division, which in a simple form expresses one or more of the equal parts into which a unit is divided.

The divisor or the number below the line in the fraction is called the **denominator** of the fraction. The denominator tells into how many parts the unit is divided.

The dividend, or the number above the line in the fraction, is called the **numerator** of the fraction. The numerator tells how many of the parts, into which the unit is divided, are taken.

The numerator and the denominator are called **the terms** of the fraction.

The **value** of a fraction is the number that it represents.

11. Mixed number.—Just as we have whole numbers and fractional numbers, so we have numbers made up of whole numbers and fractions.

Thus, we may have $2\frac{2}{3}$ which is read 2 and $\frac{2}{3}$ and means $2+\frac{2}{3}$.

Definition.—A **mixed number** is one composed of a whole number and a fraction.

12. Proper and improper fractions.—If the fraction shows fewer parts taken than the unit is divided into, its value is evidently less than 1. If the fraction shows as many or more parts taken than the unit is divided into, the fraction is evidently equal to or greater than 1.

Thus, $\frac{3}{4}$ shows fewer parts taken than the unit is divided into, and is less than 1; $\frac{4}{4}$ shows as many parts taken as the unit is divided into and is equal to 1; and $\frac{7}{4}$ shows more parts taken than the unit is divided into, and is greater than 1.

Definitions.—A **proper fraction** is one in which the numerator is less than the denominator. An **improper fraction** is one in which the numerator is equal to or greater than the denominator.

In the above, $\frac{3}{4}$ is a proper fraction, and $\frac{4}{4}$ and $\frac{7}{4}$ are improper fractions.

It should be noted that an indicated division is often called a fraction, even though the division can be performed exactly; that is, without a remainder.

Thus, $\frac{12}{3}$, $\frac{24}{12}$, $\frac{44}{11}$ are fractions.

13. Comparison of fractions.—If two fractions have equal numerators and equal denominators they are evidently equal in value.

If two fractions have **equal denominators,** the one that has the **larger numerator** is the **greater** in value. Explain why.

Thus, of $\frac{5}{7}$ and $\frac{3}{7}$, $\frac{5}{7}$ is the larger.

If two fractions have **equal numerators,** the one that has the **larger denominator** is the **smaller** in value. Explain why.

Thus, of $\frac{7}{8}$ and $\frac{7}{9}$, $\frac{7}{9}$ is the smaller.

If two fractions have **both numerators** and **denominators unequal,** their values cannot be compared so easily.

Thus, the values of $\frac{3}{4}$ and $\frac{5}{7}$ can be more easily compared when the fractions are changed to fractions that have the same denominator. See Art. 19.

14. In order to get the right viewpoint, it is well for the student to note that before he took up fractions, he had learned to add, subtract, multiply, and divide whole numbers. It is now necessary to learn how to perform the fundamental operations on fractions. They must be combined not only with other fractions but with whole numbers. The main thing in this chapter is to do these fundamental operations. But to do these in all cases it is necessary to be able to change the fractional numbers in various ways; that is, to reduce to lower or higher terms, change fractions to common denominators, mixed numbers to improper fractions, and improper fractions to mixed numbers.

In order to work efficiently when making computations and using mathematics in any practical work, one must be able to work with fractions in a ready and accurate manner. To secure the necessary skill, some students

require more drill than others. Each should take the necessary amount of practice to secure ability to work accurately and quickly.

Fractions are numbers just as much as whole numbers are, but to work with them requires special rules and methods.

15. Principles.—Since a fraction is an indicated division the following principles may be stated for fractions:

(1) *Multiplying or dividing both numerator and denominator by the same number does not change the value of the fraction.*

(2) *Multiplying the numerator or dividing the denominator by a number multiplies the fraction by that number.*

(3) *Dividing the numerator or multiplying the denominator by a number divides the fraction by that number.*

16. Reduction of a whole or a mixed number to an improper fraction.

Example 1.—Reduce 5 to 6ths.

Since

$$1=\tfrac{6}{6},\ 5=5\times\tfrac{6}{6}=\tfrac{30}{6}$$ By principle (2).

Example 2.—Reduce $7\frac{3}{5}$ to 5ths.

Since

$$1=\tfrac{5}{5},\ 7=7\times\tfrac{5}{5}=\tfrac{35}{5}$$

$$\therefore\quad 7\tfrac{3}{5}=\tfrac{35}{5}+\tfrac{3}{5}=\tfrac{38}{5}$$

The three dots $\therefore$, as used above, form a symbol meaning hence or therefore.

Rule.—*To reduce a whole number to a fraction of a given denominator, first change* 1 *to a fraction of the given denominator and then multiply the numerator by the given whole number. With a mixed number, reduce the whole number to a fraction and then add to the*

numerator of this fraction the numerator of the fractional part of the mixed number.

17. Reduction of an improper fraction to a whole or mixed number.

Example 1.—Reduce $\frac{32}{4}$ to a whole number.

$$\frac{32}{4} = 32 \div 4 = 8$$

Example 2.—Reduce $\frac{47}{9}$ to a mixed number.

$$\frac{47}{9} = 47 \div 9 = 5\frac{2}{9}$$

RULE.—*To reduce an improper fraction to a whole or mixed number, perform the indicated division. The quotient is the number of units. If there is no remainder, it reduces to a whole number. If there is a remainder, it reduces to a mixed number of which the quotient is the whole number part and the remainder the numerator of the fractional part.*

EXERCISES

1. Reduce the following numbers to 5ths: 6; 10; 16; 18; 39. To 3ds. To 10ths.

2. Reduce the following mixed numbers to improper fractions: $3\frac{1}{4}$; $8\frac{2}{5}$; $10\frac{5}{8}$; $13\frac{4}{9}$; $18\frac{3}{7}$; $19\frac{1}{9}$; $23\frac{12}{13}$; $47\frac{13}{9}$.

3. Reduce the following improper fractions to whole or mixed numbers:

$\frac{19}{2}$	$\frac{62}{8}$	$\frac{514}{18}$
$\frac{20}{6}$	$\frac{73}{3}$	$\frac{9692}{574}$
$\frac{28}{8}$	$\frac{98}{17}$	$\frac{9728}{108}$
$\frac{33}{11}$	$\frac{48}{13}$	$\frac{1010010}{555}$
$\frac{48}{7}$	$\frac{168}{28}$	$\frac{487995}{5420}$

18. Reduction of fraction to lowest terms. *Definition.*—A fraction is in its **lowest terms** when the numera-

tor and denominator are *prime* to each other; that is, when they have no common factors.

Example.—Reduce $\frac{75}{105}$ to its lowest terms

$$\frac{75}{105} = \frac{15}{21} = \frac{5}{7}$$

Since dividing both numerator and denominator by the same number does not change the value of the fraction, both terms may be divided by 5. Thus $\frac{15}{21}$ is obtained. Both terms of this fraction are divided by 3, and $\frac{5}{7}$ is obtained. Since 5 and 7 are prime to each other, the fraction is in its lowest terms. Both terms could have been divided by 15 and the reduction made in one step.

RULE.—*To reduce a fraction to its lowest terms, divide both terms successively by their common factors, or divide by the greatest common divisor of the terms.*

EXERCISES

Reduce the following fractions to their lowest terms:

1. $\frac{5}{15}$, $\frac{4}{12}$, $\frac{15}{25}$, $\frac{8}{24}$.

2. $\frac{8}{40}$, $\frac{35}{45}$, $\frac{28}{35}$, $\frac{48}{64}$.

3. $\frac{18}{26}$, $\frac{19}{38}$, $\frac{23}{43}$, $\frac{28}{49}$.

4. $\frac{74}{100}$, $\frac{20}{155}$, $\frac{17}{51}$, $\frac{100}{225}$.

5. $\frac{448}{484}$.

6. $\frac{560}{630}$.

7. $\frac{270}{360}$.

8. $\frac{288}{432}$.

9. $\frac{120}{1680}$.

10. $\frac{2252}{1008}$.

11. $\frac{231}{924}$.

12. $\frac{6030}{12060}$.

13. $\frac{3465}{4950}$.

14. $\frac{2222}{7777}$.

15. $\frac{3510}{2310}$.

16. $\frac{9378}{9873}$.

17. $\frac{78300}{837000}$.

18. $\frac{10395}{27951}$.

19. $\frac{10472}{11781}$.

20. $\frac{17703}{7266}$.

21. Reduce the following percentages to fractions in their lowest terms (the sign % takes the place of the denominator 100): 5%; 15%; 35%; 25%; 54%; 82%; 75%; 16%; 80%; 50%; 95%; 96%; 125%; 1000%; 28%.

19. Reduction of several fractions to fractions having the same denominator. *Definition.*—Fractions which

have the same denominator are called **similar** fractions or fractions with a **common denominator.**

Example 1.—Reduce $\frac{1}{2}$ and $\frac{1}{3}$ to fractions which have 6 for a denominator.

The fraction $\frac{1}{2}$ may be changed to 6ths by multiplying both its terms by a number which will make the denominator 6. This will not change the value of the fraction. This multiplier is obtained by dividing 6 by 2 which gives 3.

$$\therefore \quad \frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

Likewise

$$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

Example 2.—Reduce $\frac{7}{9}$, $\frac{3}{8}$, and $\frac{5}{6}$ to 72nds.

Both terms of $\frac{7}{9}$ are multiplied by $72 \div 9 = 8$,
both terms of $\frac{3}{8}$ are multiplied by $72 \div 8 = 9$,
both terms of $\frac{5}{6}$ are multiplied by $72 \div 6 = 12$.

$$\therefore \quad \frac{7}{9} = \frac{56}{72}, \ \frac{3}{8} = \frac{27}{72}, \text{ and } \frac{5}{6} = \frac{60}{72}$$

RULE.—*To reduce several fractions to fractions having a common denominator, multiply both terms of each fraction by a number found by dividing the common denominator by the denominator of that fraction.*

Fractions which have the same denominators are usually more easily compared as to size than are fractions with different denominators. Thus, one can as readily tell which is the larger of $\frac{4}{7}$ and $\frac{6}{7}$ as he can tell how six apples compare with four apples; but $\frac{2}{7}$ and $\frac{4}{13}$ are not so easily compared in size. However, if these are reduced to common denominators, they become $\frac{26}{91}$ and $\frac{28}{91}$ and can readily be compared.

20. Least common denominator.—In the preceding article the common denominator 72 was given. Usually the denominator is not given, but we are asked to reduce the given fractions to fractions having a least common denominator. When this is the case, we find the least common multiple of the denominators of the given fractions, and this is the **least common denominator** (L. C. D.) for all the fractions.

Example 1.—Reduce $\frac{4}{9}$, $\frac{7}{12}$, and $\frac{13}{24}$ to fractions with a L. C. D.

The L. C. M. of 9, 12, and 24 is 72. If we divide 72 by each of the given denominators we get the numbers to be used as multipliers.

$\therefore \quad \frac{4}{9} = \frac{32}{72}$, $\frac{7}{12} = \frac{42}{72}$, and $\frac{13}{24} = \frac{39}{72}$

Remark.—Usually the fractions dealt with have such denominators that their L. C. D. can be seen by inspection. The student should endeavor to determine it in this way wherever possible. If it cannot be seen by inspection, a good way to find it is as follows: Divide the given denominators by a *prime* number that will divide *two* or *more* of them, then divide the remaining numbers and the quotients by a prime number that will divide two or more of them. Continue this as long as possible. The L. C. D. is the continued product of all the divisors and the quotients of numbers left.

Example 2.—Find the L. C. D. of $\frac{11}{30}$, $\frac{7}{45}$, $\frac{14}{135}$, and $\frac{13}{25}$.

Process

5)30, 45, 135, 25
3)6, 9, 27, 5
3)2, 3, 9, 5
2, 1, 3, 5

L. C. D. $= 5 \times 3 \times 3 \times 2 \times 3 \times 5 = 1350$.

EXERCISES

Change as indicated:

1. $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$ to 12ths.
2. $\frac{1}{2}$, $\frac{1}{6}$, and $\frac{1}{7}$ to 42ds.
3. $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{5}{6}$ to 30ths.
4. $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to 24ths.
5. 3, $\frac{3}{8}$, and $\frac{7}{9}$ to 72ds.
6. 4, 5, $\frac{3}{4}$, and $\frac{4}{5}$ to 20ths.
7. $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{6}$, and $\frac{7}{15}$ to 90ths.
8. $\frac{3}{5}$, $\frac{7}{4}$, $\frac{11}{20}$, and $\frac{15}{50}$ to 100ths.
9. $\frac{13}{16}$, $\frac{9}{11}$, $\frac{5}{8}$, and $\frac{1}{4}$ to 176ths.
10. $\frac{11}{24}$, $\frac{13}{17}$, $\frac{5}{6}$, and $\frac{2}{3}$ to 408ths.

Change the following to fractions having a L. C. D.:

11. $\frac{1}{2}$ and $\frac{3}{7}$.
12. $\frac{5}{6}$ and $\frac{14}{15}$.
13. $\frac{37}{39}$ and $\frac{11}{9}$.
14. $\frac{5}{7}$ and $\frac{11}{13}$.
15. $\frac{6}{11}$ and $\frac{13}{17}$.
16. $\frac{11}{17}$ and $\frac{13}{19}$.
17. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{77}{824}$.
18. $\frac{11}{30}$, $\frac{13}{60}$, $\frac{4}{81}$, and $\frac{35}{18}$.
19. $3\frac{5}{8}$, $5\frac{13}{12}$, and $7\frac{9}{40}$.

20. Change the following to 100ths and then write as percents:

$\frac{1}{2}$	$\frac{1}{10}$	$\frac{19}{20}$	$\frac{7}{50}$
$\frac{1}{4}$	$\frac{7}{10}$	$\frac{1}{25}$	$\frac{13}{50}$
$\frac{3}{4}$	$\frac{9}{10}$	$\frac{3}{25}$	$\frac{47}{50}$
$\frac{1}{5}$	$\frac{1}{20}$	$\frac{21}{25}$	$\frac{77}{50}$
$\frac{3}{5}$	$\frac{7}{20}$	$\frac{23}{25}$	$\frac{98}{50}$
$\frac{6}{5}$	$\frac{13}{20}$	$\frac{1}{50}$	$\frac{125}{50}$

21. Which is the larger: $\frac{5}{8}$ or $\frac{11}{18}$? $\frac{2}{3}$ or $\frac{11}{16}$? $\frac{7}{11}$ or $\frac{20}{31}$?

22. A number of drills are, respectively, $\frac{5}{8}$; $\frac{11}{16}$; $\frac{1}{4}$; $\frac{19}{32}$; $\frac{23}{64}$; $\frac{13}{32}$; $\frac{27}{64}$; $\frac{1}{2}$; $\frac{17}{64}$; $\frac{3}{8}$; and $\frac{33}{64}$ inches in diameter. Reduce the sizes to 64ths and arrange in order of size, with smallest first.

ADDITION OF FRACTIONS

21. *Example* 1.—Add $\frac{7}{12}$, $\frac{5}{12}$, and $\frac{11}{12}$.

Just as 7 apples + 5 apples + 11 apples = 23 apples, so 7 twelfths + 5 twelfths + 11 twelfths = 23 twelfths.

The work may be arranged as follows:

$$\frac{7}{12}+\frac{5}{12}+\frac{11}{12}=\frac{23}{12}=1\frac{11}{12}$$

Example 2.—Find sum of $\frac{7}{12}$, $\frac{8}{15}$, and $\frac{17}{30}$.

Here the fractions must first be reduced to fractions having a L. C. D. The L. C. M. of 12, 15, and 30 is 60.

Then

$$\frac{7}{12}+\frac{8}{15}+\frac{17}{30}=\frac{35}{60}+\frac{32}{60}+\frac{34}{60}=\frac{101}{60}=1\frac{41}{60}$$

Example 3.—Find the sum of $3\frac{3}{4}$, $5\frac{4}{7}$, $2\frac{9}{14}$, and $7\frac{1}{2}$.

The whole numbers and the fractions may be added separately and then these sums united. The work may be written as here:

$$3\frac{3}{4}+5\frac{4}{7}+2\frac{9}{14}+7\frac{1}{2}=3\frac{21}{28}+5\frac{16}{28}+2\frac{18}{28}+7\frac{14}{28}=17\frac{69}{28}=19\frac{13}{28}.$$

A more convenient way of writing the mixed numbers for adding, is to write them under each other, and add, similar to the method of adding whole numbers.

$$\begin{array}{rcl} 3\frac{3}{4} & = & 3\frac{21}{28} \\ 5\frac{4}{7} & = & 5\frac{16}{28} \\ 2\frac{9}{14} & = & 2\frac{18}{28} \\ 7\frac{1}{2} & = & 7\frac{14}{28} \\ \hline & & 17\frac{69}{28}=19\frac{13}{28} \end{array}$$

RULE.—*To add fractions that have a L. C. D., add the numerators of the fractions and place the sum over the L. C. D. If this gives an improper fraction, it should be reduced to a whole or mixed number. If the fractions do not have a L. C. D., first reduce them to fractions with a*

L. C. D. To add mixed numbers, add the whole numbers and fractions separately and then unite the sums.

EXERCISES

Add the following and express the sum in the simplest form:

1. $\frac{3}{7}+\frac{8}{7}+\frac{11}{7}+\frac{4}{7}$.
2. $\frac{2}{9}+\frac{5}{9}+\frac{19}{9}+\frac{23}{9}$.
3. $\frac{5}{14}+\frac{9}{14}+\frac{27}{14}+\frac{11}{14}$.
4. $\frac{4}{5}+\frac{5}{6}+\frac{7}{10}$.
5. $8+\frac{7}{4}+\frac{8}{3}+\frac{7}{2}$.
6. $9\frac{1}{6}+8\frac{5}{3}+7$.
7. $14\frac{3}{4}+30\frac{1}{2}+4$.
8. $7\frac{2}{3}+9\frac{3}{4}+11\frac{1}{2}$.
9. $\frac{9}{10}+\frac{7}{12}+\frac{5}{4}+\frac{2}{3}$.
10. $\frac{7}{2}+\frac{2}{7}+\frac{9}{14}+\frac{11}{28}$.
11. $\frac{222}{3}+\frac{333}{2}+\frac{100}{6}$.
12. $\frac{13}{7}+\frac{23}{4}+\frac{111}{14}$.
13. $\frac{123}{3}+231\frac{7}{32}+321\frac{5}{8}$.
14. $5\frac{2}{3}+71\frac{3}{4}+82\frac{11}{12}+5\frac{5}{6}$.

15. $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}+\frac{7}{8}+\frac{11}{12}+\frac{13}{12}$.
16. $2\frac{2}{3}+5\frac{3}{4}+7\frac{5}{6}+11\frac{11}{12}+13\frac{13}{12}$.
17. $135\frac{17}{130}+531\frac{7}{13}+315\frac{19}{10}$.
18. $147\frac{5}{9}+61\frac{7}{2}+\frac{22}{81}+258+\frac{73}{18}$.
19. $61\frac{3}{5}+16\frac{11}{15}+41\frac{2}{3}+14\frac{13}{15}+77\frac{7}{3}+55\frac{2}{5}$.
20. $12\frac{1}{12}+24\frac{1}{24}+36\frac{1}{36}+48\frac{1}{48}+60\frac{1}{60}$.
21. $11\frac{2}{3}+13\frac{11}{21}+15\frac{5}{7}+17\frac{17}{21}+19\frac{19}{3}$.

22. A golf player drives a golf ball $253\frac{3}{4}$ yards, $189\frac{9}{10}$ yards, $33\frac{7}{8}$ yards, $5\frac{1}{2}$ yards, and $1\frac{1}{5}$ yards. Find the total number of yards that the ball was driven.

23. At the end of each hour for 5 hours an airplane was noted to have flown $117\frac{5}{6}$, $166\frac{7}{8}$, $213\frac{5}{16}$, $258\frac{2}{3}$, and 160 miles, respectively. What was the total distance flown in 5 hours?

24. A merchant sold to different customers $4\frac{1}{4}$, $7\frac{2}{3}$, $13\frac{5}{8}$, $15\frac{3}{4}$, and $2\frac{1}{2}$ yards of cloth, respectively. Find the total number yards sold.

25. A brass rod was cut into five pieces of lengths $4\frac{1}{4}$, $3\frac{5}{8}$, $6\frac{1}{2}$, $7\frac{9}{16}$, and $2\frac{3}{4}$ inches, respectively. How long was the rod if $\frac{1}{16}$ inch was wasted in each cut?

SUBTRACTION OF FRACTIONS

22. *Example* 1.—Subtract $\frac{4}{11}$ from $\frac{9}{11}$.

Since like members can be subtracted we can subtract 4 elevenths from 9 elevenths and have the remainder 5 elevenths. This may be written $\frac{9}{11}-\frac{4}{11}=\frac{5}{11}$.

Example 2.—Subtract $\frac{7}{11}$ from $\frac{2}{3}$.

Here the fractions must first be reduced to fractions having the same denominator. It may be written

$$\frac{2}{3}-\frac{7}{11}=\frac{2}{3}\frac{2}{3}-\frac{2}{3}\frac{1}{3}=\frac{1}{3}\frac{}{3}$$

Example 3: $7\frac{2}{3}-3\frac{3}{5}=$ what?

Solution

$$7\frac{2}{3}=7\frac{10}{15}$$
$$3\frac{3}{5}=3\frac{9}{15}$$
$$4\frac{1}{15}$$

In this case the fractional part of the subtrahend is less than that of the minuend. The fractional parts of mixed numbers are reduced to fractions having the L. C. D., the fractional parts subtracted, and then the whole numbers subtracted.

Example 4: $7\frac{1}{2}-3\frac{2}{3}=$ what?

Solution

$$7\tfrac{1}{2}=7\tfrac{3}{6}=6\tfrac{9}{6}$$
$$3\tfrac{2}{3}=3\tfrac{4}{6}=3\tfrac{4}{6}$$
$$3\tfrac{5}{6}$$

In this case the fractional part of the subtrahend is greater than the fractional part of the minuend. The fractions are changed to fractions having the L. C. D. as before. It is then noticed that the fraction $\frac{1}{6}$ in the

subtrahend is larger than $\frac{3}{6}$ in the minuend and so cannot be subtracted from it. To overcome this difficulty, we take 1 from the 7 and change it to sixths. This gives $6\frac{9}{6}$ instead of $7\frac{3}{6}$. The subtraction is then made as before.

RULE.—*To find the difference between two fractions having a common denominator, find the difference of the numerators and write it over the common denominator. If the fractions do not have a L. C. D., reduce them to such before subtracting. If the numbers are mixed numbers, subtract the fractional parts and then the whole numbers.*

EXERCISES

Subtract the following and give the results in the simplest forms:

1. $\frac{2}{3}-\frac{1}{3}$.
2. $\frac{2}{3}-\frac{1}{6}$.
3. $\frac{7}{8}-\frac{3}{4}$.
4. $\frac{9}{10}-\frac{3}{5}$.
5. $\frac{11}{16}-\frac{1}{2}$.
6. $3\frac{1}{3}-2\frac{2}{3}$.
7. $7\frac{3}{5}-3\frac{9}{10}$.
8. $13-12\frac{11}{12}$.
9. $16-14\frac{23}{32}$.
10. $\frac{47}{18}-\frac{71}{90}$.
11. $\frac{19}{131}-\frac{2}{19}$.
12. $\frac{111}{316}-\frac{31}{72}$.
13. $6\frac{2}{9}-2\frac{17}{90}$.
14. $77\frac{7}{8}-11\frac{8}{9}$.
15. $18\frac{91}{100}-15$.

16.
$643\frac{3}{4}$
$346\frac{7}{8}$

17.
$6491\frac{19}{33}$
$491\frac{10}{11}$

18.
$320\frac{7}{8}$
$230\frac{2}{3}$

19.
$309\frac{5}{6}$
$190\frac{3}{4}$

20.
$189\frac{7}{17}$
$101\frac{7}{34}$

21.
$171\frac{5}{8}$
$17\frac{2}{3}$

22.
$164\frac{3}{10}$
$155\frac{3}{8}$

23.
$2929\frac{29}{39}$
$929\frac{11}{78}$

24.
$1111\frac{11}{10}$
$1001\frac{11}{30}$

Simplify the following, that is, do the operations indicated:

25. $21\frac{3}{11}+19\frac{2}{7}-10\frac{13}{28}$.
26. $6\frac{6}{7}+5\frac{13}{20}-10$.
27. $7\frac{3}{4}-\frac{4}{5}+2\frac{17}{20}$.
28. $5\frac{12}{13}-\frac{5}{26}-\frac{11}{13}+19$.
29. $13\frac{3}{5}+\frac{7}{8}-7\frac{3}{4}-\frac{1}{8}+2$.
30. $13-(6\frac{7}{8}-\frac{3}{8})+\frac{1}{2}-\frac{5}{8}$.

The parentheses indicate that the enclosed operations should be performed first. Thus, in the above, $\frac{3}{8}$ should be subtracted from $6\frac{7}{8}$ before they are subtracted from 13.

31. $\frac{7}{8}+78+\frac{3}{3}\frac{1}{2}-(6+7\frac{8}{9})$. **34.** $\frac{3}{8}+\frac{3}{7}+21-(\frac{5}{8}-\frac{1}{2}\frac{1}{1})$.

32. $1\frac{2}{3}+4\frac{5}{6}+7\frac{8}{9}-(\frac{1}{1}\frac{7}{8}+\frac{1}{3})$. **35.** $43+(10\frac{1}{2}\frac{8}{1}-1\frac{1}{7}-\frac{3}{1}\frac{}{4})$.

33. $2\frac{3}{4}+5\frac{6}{7}-(1\frac{1}{2}-1\frac{1}{7})+13\frac{1}{4}$.

Do as many of the following as you can without a pencil:

36. A man had $\frac{6}{7}$ of a debt to pay and paid $\frac{1}{2}$ of it. How much did he have left to pay?

37. A man had $\frac{3}{4}$ of a dollar and lost $\frac{7}{1}\frac{}{0}$ of it. How much money did he have left?

38. It takes $21\frac{1}{1}\frac{}{2}$ days to do a job. If a man works $10\frac{1}{2}$ days at the job; how much is left to do?

39. A man can do a piece of work in $13\frac{7}{8}$ days. A boy can do the same piece of work in $19\frac{1}{2}$ days. How much longer does it take the boy to do the work?

40. A boy owed his friend $\$2\frac{3}{4}$. One day he paid $\$\frac{4}{5}$, and another day he paid $\$\frac{3}{1}\frac{}{0}$. How much is left to pay?

41. A farmer had $36\frac{3}{4}$ dozen eggs to sell. To one store he sold $13\frac{1}{2}$ dozen; to another store he sold 16 dozen; and the remainder to a third store. How many eggs did the third store buy?

42. A dealer had 16 gallons of oil to sell. He sold $1\frac{1}{2}$ gallons to one customer, $2\frac{3}{4}$ gallons to another, $7\frac{1}{4}$ gallons to another, and the remainder to a fourth customer. How much did he sell to the fourth customer?

43. A tank full of water has two pipes opening from it, one will empty $\frac{1}{4}$ of the water in the tank in one hour and the other $\frac{7}{1}\frac{}{2}$ of it. What part will both pipes empty in 1 hour? What part remains in the tank? Suppose the two pipes run for 1 hour; the first one filling the tank, and the second emptying it, how much will be left in the tank?

44. Find the distance around Fig. 3 with dimensions as given.

45. Find the number of feet of molding to extend around the room in Fig. 4 if no allowance is made for waste.

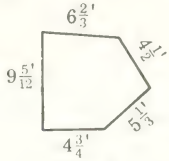

FIG. 3.

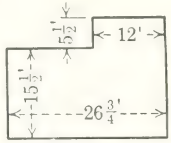

FIG. 4.

46. A man had $3\frac{2}{3}$ tons of coal delivered onMonday, $6\frac{7}{8}$ tons delivered on Tuesday, $1\frac{1}{2}$ tons delivered on Wednesday, $5\frac{11}{12}$ tons delivered on Thursday, and $3\frac{1}{3}$ tons delivered on Friday. How much should be delivered on Saturday if the total amount delivered the first 3 days is the same as the amount delivered the last 3 days?

47. The distance from outside to outside between two holes in a steel plate is $6\frac{2}{3}$ inches. If one hole is $1\frac{1}{6}$ inches in diameter and the other is $2\frac{1}{4}$ inches in diameter, find the length of metal between the holes.

MULTIPLICATION OF FRACTIONS

23. Multiplication of a fraction and an integer.

Example 1.—Multiply $\frac{3}{5}$ by 4.

To multiply $\frac{3}{5}$ by 4 is to find a fraction that is 4 times as large as $\frac{3}{5}$. By Art. 14, multiplying the numerator of a fraction multiplies the value of the fraction.

$$\therefore \quad \frac{3}{5}\times 4=\frac{3\times 4}{5}=\frac{12}{5}=2\frac{2}{5}$$

Example 2.—Multiply 8 by $\frac{2}{3}$.

Since in finding the product of two numbers either may be used for the multiplier without changing the product,

$$\therefore \quad 8\times\frac{2}{3}=\frac{2}{3}\times 8=\frac{16}{3}=5\frac{1}{3}$$

Example 3.—Multiply $\frac{3}{14}$ by 7.

Here we may use the principle that dividing the denominator multiplies the value of the fraction, or the operation may be thought of as one in cancellation.

$$\therefore \quad \frac{3}{14}\times 7 = \frac{3}{14\div 7} = \frac{3}{2} = 1\frac{1}{2}$$

Or

$$\frac{3}{14}\times 7 = \frac{3\times 7}{14} = \frac{3}{2} = 1\frac{1}{2}$$

Here the 7 and 14 are canceled.

RULE.—*To multiply a fraction by an integer or an integer by a fraction, multiply the numerator or divide the denominator of the fraction by the integer.*

The denominator should be divided when possible because then the result is in lower terms than there would be if the numerator were multiplied.

Thus, in example 3, if the numerator is multiplied, we have $\frac{3}{14}\times 7 = \frac{21}{14}$ instead of $\frac{3}{2}$ which we get when the denominator is divided.

Remark.—When a whole number is multiplied by a whole number, the product is larger than the multiplicand; but whenever the multiplier is a proper fraction, the product is smaller than the multiplicand. Here we cannot think of multiplication as a shortened addition, as we can when multiplying by a whole number.

We often write $\frac{2}{3}$ of 6 for $\frac{2}{3}\times 6$. The meaning is the same in each case.

24. Multiplication of a fraction by a fraction.

Example 1.—Multiply $\frac{2}{3}$ by $\frac{5}{7}$.

$\frac{2}{3}$ of $\frac{5}{7}$ is the same as $\frac{5}{7}$ of $\frac{2}{3}$; but $\frac{5}{7}$ of $\frac{2}{3}$ is 5 times $\frac{1}{7}$ of $\frac{2}{3}$, and $\frac{1}{7}$ of $\frac{2}{3}$ has a value $\frac{1}{7}$ as large as $\frac{2}{3}$.

By Art. 15, the value of a fraction is divided when the denominator is multiplied.

$$\therefore \qquad \tfrac{1}{7} \text{ of } \tfrac{2}{3} = \frac{2}{3 \times 7} = \tfrac{2}{21}$$

And

$$\tfrac{5}{7} \text{ of } \tfrac{2}{3} = 5 \text{ times } \tfrac{2}{21} = \frac{2 \times 5}{21} = \tfrac{10}{21}$$

These steps may be combined as follows:

$$\tfrac{2}{3} \times \tfrac{5}{7} = \frac{2 \times 5}{3 \times 7} = \tfrac{10}{21}$$

Example 2.—Multiply $\frac{14}{18}$ by $\frac{3}{7}$.

$$\tfrac{14}{18} \times \tfrac{3}{7} = \frac{\overset{2}{\cancel{14}} \times \overset{1}{\cancel{3}}}{\underset{6}{\cancel{18}} \times 7} = \tfrac{1}{3}$$

Cancellation should be used when it will shorten the work.

Example 3.—Multiply $\frac{3}{5}$ by $\frac{9}{13}$ by $\frac{26}{27}$.

$$\tfrac{3}{5} \times \tfrac{9}{13} \times \tfrac{26}{27} = \frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{9}} \times \overset{2}{\cancel{26}}}{5 \times \underset{1}{\cancel{13}} \times \underset{3}{\cancel{27}}} = \tfrac{2}{5}$$

RULE.—*To multiply a fraction by a fraction, multiply the numerators together for the numerator of the product, and the denominators together for the denominator of the product. Cancel when convenient.*

Remark.—A form like $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{8}$ is often called a **compound fraction.**

25. Multiplication of mixed numbers.

Example 1.—Multiply $7\frac{3}{5}$ by 6.

$$7\tfrac{3}{5}\times 6=\frac{38}{5}\times 6=\frac{228}{5}=45\tfrac{3}{5}$$

Example 2.—Multiply $8\frac{1}{3}$ by $3\frac{2}{5}$.

$$8\tfrac{1}{3}\times 3\tfrac{2}{5}=\frac{\overset{5}{\cancel{25}}}{\underset{3}{\cancel{3}}}\times\frac{17}{\underset{5}{\cancel{5}}}=\frac{85}{3}=28\tfrac{1}{3}$$

RULE.—*To multiply two numbers, one or both of which are mixed numbers, reduce the mixed numbers to improper fractions and multiply as with fractions.*

Remark.—The work may often be simplified by using the following methods:

Example 3.—Multiply 47 by $16\frac{4}{5}$.

Process

$$\begin{array}{r} 47 \\ 16\frac{4}{5} \\ \hline 5)188 \\ \hline 37\frac{3}{5} \\ 282 \\ 47 \\ \hline 789\frac{3}{5} \end{array}$$

Explanation.—Multiply 47 by 4 and divide by 5, which is the same as multiplying 47 by $\frac{4}{5}$; this gives $37\frac{3}{5}$. Then multiply 47 by 16 in the ordinary way for multiplying whole numbers. Add these three partial products and the entire product is $789\frac{3}{5}$.

Example 4.—Multiply $25\frac{2}{5}$ by $6\frac{1}{3}$.

Process

$$\begin{array}{r} 25\frac{2}{5} \\ 6\frac{1}{3} \\ \hline \frac{2}{15} \\ 8\frac{1}{3} \\ 2\frac{2}{5} \\ 150 \\ \hline 160\frac{13}{15} \end{array}$$

Explanation: $\frac{2}{5}\times\frac{1}{3}=\frac{2}{15}$; $25\times\frac{1}{3}=8\frac{1}{3}$; $\frac{2}{5}\times 6=2\frac{2}{5}$; $25\times 6=150$.

The entire product equals the sum of these partial products.

If several fractions and mixed numbers are to be multiplied together it is usually best to reduce all to fractions for then the work may be shortened by cancellation.

Example 5.—Find the product of $\frac{7}{9}\times 3\frac{2}{3}\times 9\times 4\frac{2}{11}\times\frac{3}{22}$.

$$\frac{7}{9}\times 3\frac{2}{3}\times 9\times 4\frac{2}{11}\times\frac{3}{22}=\frac{7}{\cancel{9}}\times\frac{\overset{11}{\cancel{11}}}{\cancel{3}}\times\frac{\cancel{9}}{1}\times\frac{\overset{23}{\cancel{46}}}{\cancel{11}}\times\frac{\cancel{3}}{\underset{11}{\cancel{22}}}=\frac{161}{11}=14\frac{7}{11}$$

EXERCISES

Find the product of each of the following without using a pencil when possible:

1. $\frac{3}{4}\times 5$.	**7.** $11\times\frac{2}{3}$.	**13.** $21\times\frac{5}{7}$.
2. $\frac{2}{5}\times 10$.	**8.** $7\times\frac{3}{4}$.	**14.** $35\times\frac{3}{5}$.
3. $\frac{7}{8}\times 3$.	**9.** $12\times\frac{3}{4}$.	**15.** $66\times 2\frac{1}{6}$.
4. $\frac{5}{9}\times 9$.	**10.** $36\times\frac{7}{9}$.	**16.** $55\times 4\frac{1}{5}$.
5. $\frac{7}{20}\times 5$.	**11.** $18\times\frac{2}{17}$.	**17.** $63\times 2\frac{2}{9}$.
6. $\frac{3}{20}\times 8$.	**12.** $7\times\frac{3}{13}$.	**18.** $\frac{6}{7}\times\frac{7}{6}$.

19. $\frac{2}{3}$ of 9. **22.** $\frac{9}{10}$ of 110. **25.** $7\frac{7}{15}\times 75$.

20. $\frac{3}{4}$ of 20. **23.** $\frac{7}{8}$ of 100. **26.** $8\frac{2}{3}\times 12$.

21. $\frac{5}{9}$ of 63. **24.** $\frac{1}{20}$ of 80. **27.** $5\frac{7}{12}\times 6$.

28. What is 5 times 6 cents? 5 times 6-sevenths? 5 times $\frac{7}{8}$? $5\times\frac{9}{10}$?

29. What is $\frac{1}{4}$ of \$12? $\frac{1}{4}$ of 8-tenths? $\frac{1}{4}$ of $\frac{8}{11}$? $\frac{1}{4}\times\frac{8}{15}$?

30. A tank is $\frac{7}{8}$ full of gas. If $\frac{1}{5}$ of this is drawn off, what part of the whole tank is drawn off? What part remains in the tank?

31. One man paid \$12,650 for a home and another man paid $\frac{4}{5}$ as much. How much did the second man pay for his home?

32. A tank holds 300 gallons of gas. If a pipe empties $\frac{1}{4}$ of the gas in an hour, how many gallons will be left in the tank at the end of two hours?

33. A boy had \$48$\frac{1}{2}$. He spent $\frac{1}{2}$ of this for a suit and $\frac{1}{2}$ of the remainder for a gun. How much did he have left?

34. The circumference of a circle is about $3\frac{1}{7}$ times the diameter. Find the circumference of a circle if the diameter is 14 feet; if 28 feet; if $\frac{1}{22}$ foot.

35. The diagonal of a square is very nearly $1\frac{5}{12}$ the length of one side. Find the diagonal when one side is 24 inches; when 18 inches; when 840 feet.

36. If you have \$250 to spend and spend $\frac{3}{5}$ of it for camping, $\frac{1}{10}$ of it for a hat, and the remainder for traveling, how much did you spend for each?

37. A child went to the store with \$7 and 20 cents. He spent $\frac{1}{3}$ of his money for skates, $\frac{3}{5}$ of it for a coat, and $\frac{1}{9}$ of it for a book. How much did he spend for each article? How much did he have left?

38. $\frac{35}{169}\times 13$. **42.** $618\frac{5}{6}\times 60$. **46.** $24\frac{7}{8}\times 40\frac{2}{3}$.

39. $\frac{63}{72}\times 108$. **43.** $569\frac{2}{5}\times 15$. **47.** $21\frac{7}{9}\times 81\frac{3}{7}$.

40. $\frac{50}{25}\frac{80}{25}\times 625$. **44.** $11\frac{2}{5}\times 12\frac{2}{5}$. **48.** $16\frac{2}{3}\times 18\frac{3}{4}$.

41. $32\frac{5}{9}\times 18$. **45.** $12\frac{2}{3}\times 15\frac{2}{5}$. **49.** $360\frac{7}{12}\times 144\frac{3}{10}$.

50. Multiply $1\frac{1}{2}$ by $1\frac{1}{2}$, $2\frac{1}{2}$ by $2\frac{1}{2}$, $3\frac{1}{2}$ by $3\frac{1}{2}$, $10\frac{1}{2}$ by $10\frac{1}{2}$.

51. Can you make a rule for finding the product of two factors that are the same and end in $\frac{1}{2}$? See Art. 48 (5).

The product of two factors that are exactly alike is called the *square* of one of them. Thus, the square of $4\frac{1}{2}$ is $4\frac{1}{2}\times 4\frac{1}{2}=20\frac{1}{4}$.

Find the square of each of the following by your rule: $7\frac{1}{2}$, $9\frac{1}{2}$, $11\frac{1}{2}$, $16\frac{1}{2}$, $12\frac{1}{2}$, $20\frac{1}{2}$, $100\frac{1}{2}$.

52. $\frac{2}{5}\times\frac{5}{6}\times\frac{9}{10}=$ what? **53.** $7\frac{2}{3}\times\frac{6}{7}\times\frac{3}{8}\times\frac{7}{23}=$ what?

54. $\frac{2}{3}\times\frac{7}{8}\times\frac{4}{5}\times\frac{6}{7}=$ what?

55. $\frac{11}{12}\times 3\frac{3}{4}\times 5\frac{1}{6}\times\frac{10}{33}\times 7\frac{4}{5}=$ what?

56. If butter is worth $24\frac{4}{5}$ cents a pound, what is $20\frac{1}{2}$ pounds worth?

57. If a motor makes 2100 R. P. M., how many revolutions does it make in $\frac{3}{4}$ hour? In $3\frac{10}{21}$ days?

58. Is it cheaper to hire a boy at $35\frac{4}{5}$ cents per hour for $10\frac{5}{8}$ hours to do a piece of work or a man at $95\frac{1}{2}$ cents per hour who can do the work in 4 hours?

59. A company hires 12 men at \$1 and $44\frac{1}{2}$ cents per hour, 6 at \$1 and $96\frac{2}{3}$ cents per hour, and 10 girls at $90\frac{1}{5}$ cents per hour. They work 8 hours a day for $5\frac{1}{2}$ days a week. What is the weekly pay roll?

60. A man drives $8\frac{1}{3}$ hours at an average rate of $38\frac{9}{10}$ miles per hour. If he keeps driving at this rate for $3\frac{1}{2}$ days of $8\frac{1}{3}$ hours per day, what will be the total distance driven?

61. A gang of men mix and place an average of $38\frac{2}{2}\frac{1}{5}$ cubic yards of concrete an hour. How many cubic yards do they place in a day of $7\frac{5}{6}$ hours?

62. An alloy, used for bearings in machinery, is $\frac{24}{29}$ copper, $\frac{4}{29}$ tin, and $\frac{1}{29}$ zinc. How many pounds of each in 346 pounds of the alloy?

63. An alloy, called antifriction metal, is $\frac{370}{1000}$ copper, $\frac{115}{125}$ tin, and $\frac{3}{40}$ antimony. Find the weight of each metal in a mass of the alloy weighing 1250 pounds.

64. Find the cost of $32\frac{1}{9}$ square yards of floor covering if one square yard costs $\$2\frac{3}{4}$.

65. A boy had $\$8\frac{11}{20}$. He spent $\frac{2}{5}$ of it for carfare and lost $\frac{1}{3}$ of the remainder. How much did he spend? How much did he lose? How much did he bring home?

66. The velocity of sound in air at 0 degrees centigrade and 76 centimeters pressure is $1086\frac{7}{10}$ feet per second, and the velocity increases $5\frac{1}{16}$ foot per second per degree rise in temperature. Find the velocity of sound in air that is 30 degrees centigrade. The velocity of sound in water is about $4\frac{1}{3}$ times the velocity of sound in air. What is the velocity of sound in water at 30 degrees centigrade?

67. An article costs $\$16\frac{13}{20}$ to manufacture. It was found that $\frac{5}{8}$ of the cost was for labor, $\frac{3}{16}$ was for materials, and the remainder for incidentals. What was the amount of each separate cost?

68. If a city block is $\frac{1}{8}$ mile, how far has a man walked when he has gone 5 blocks east and $11\frac{1}{2}$ blocks north?

69. A company employs 165 men at $\$2\frac{1}{5}$ per hour, 228 men at $\$1\frac{3}{4}$ per hour, 560 men at $\$1\frac{1}{20}$ per hour, 105 men at $\$\frac{2}{5}$ per hour. What would be the total amount of the pay roll for 1 week of $5\frac{1}{2}$ days of $6\frac{1}{2}$ hours each?

70. Remembering that 6% (6 per cent) means $\frac{6}{100}$, find the value of the following:

(1) 6% of a note of $1200.
(2) 3% of the cost of a suit that costs $22.
(3) 20% of a class of 40 students.
(4) 12% of 600 dozen eggs.
(5) 75% of a distance of 132 miles.
(6) $12\frac{1}{2}$% of a gang of 64 men.
(7) $33\frac{1}{3}$% of a tuition fee of $150.
(8) 91% of a weight of 3275 pounds.

DIVISION OF FRACTIONS

26. Division of a fraction by an integer.

Example 1.—Divide $\frac{3}{7}$ by 4.

(1) Since to divide by 4 is to find one of the 4 equal parts, and to get $\frac{1}{4}$ of a number is to find one of the 4 equal parts, we have

$$\tfrac{3}{7} \div 4 = \tfrac{1}{4} \text{ of } \tfrac{3}{7} = \tfrac{3}{28}$$

(2) Or, using the principle that multiplying the denominator of a fraction divides the value of the fraction, we have

$$\tfrac{3}{7} \div 4 = \frac{3}{7 \times 4} = \tfrac{3}{28}$$

(3) In division of fractions, we can often divide the numerator and thus divide the fraction.

Example 2.—Divide $\frac{925}{11}$ by 25.

$$\frac{925}{11} \div 25 = \frac{925 \div 25}{11} = \frac{37}{11} = 3\tfrac{4}{11}$$

This may be written $\frac{925}{11} \div 25 = \frac{925}{11} \times \frac{1}{25} = \frac{37}{11} = 3\frac{4}{11}$.

RULE.—*To divide a fraction by an integer, divide the numerator, or multiply the denominator, of the fraction by the integer; or multiply the fraction by* 1 *over the integer.*

27. Division by a fraction.

Example 1.—Divide 6 by $\frac{2}{3}$.

(1) If we reduce 6 to thirds, we may divide the numerators, since then the numbers will both be thirds, and so be *like* numbers.

$$\therefore \quad 6 \div \tfrac{2}{3} = \tfrac{18}{3} \div \tfrac{2}{3} = 18 \text{ thirds} \div 2 \text{ thirds} = 9$$

(2) Since there are 3 times $\frac{1}{3}$ in 1, and $\frac{1}{2}$ as many times $\frac{2}{3}$, there are $\frac{1}{2}$ times 3, or $\frac{3}{2}$, times $\frac{2}{3}$ in 1. Now $\frac{3}{2}$ is $\frac{2}{3}$ inverted. Hence we can find how many times the fraction $\frac{2}{3}$ is contained in 1 by inverting the fraction. $\frac{2}{3}$ will be contained 6 times as many times in 6 as it is contained in 1.

$$\therefore \quad 6 \div \tfrac{2}{3} = 6 \times \tfrac{3}{2} = 9$$

Definition.—The **reciprocal** of a number is 1 divided by that number. Thus, $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$. $\frac{1}{4}$ is the reciprocal of $\frac{4}{1}$ or 4.

Example 2.—Divide $\frac{49}{65}$ by $\frac{14}{39}$.

$$\frac{49}{65} \div \frac{14}{39} = \frac{49}{65} \times \frac{39}{14} = \frac{21}{10} = 2\frac{1}{10}$$

Example 3.—Divide $4\frac{4}{5}$ by $3\frac{1}{3}$.

First reducing each to improper fractions, we have

$$4\frac{4}{5} \div 3\frac{1}{3} = \frac{24}{5} \div \frac{10}{3} = \frac{24}{5} \times \frac{3}{10} = \frac{36}{25} = 1\frac{11}{25}$$

RULE.—*To divide a whole number or a fraction by a fraction, invert the divisor and multiply by the dividend. If either or both dividend and divisor are mixed numbers, first change to improper fractions.*

28. Special methods in division.—The work of division may often be simplified by one of the following methods:

Example 1.—Divide $56\frac{2}{3}$ by 5.

Process

$$5)56\frac{2}{3}$$
$$11\frac{1}{3}$$

Explanation: $56 \div 5 = 11$ with a remainder of 1. $1\frac{2}{3} \div 5 = \frac{5}{3} \div 5 = \frac{1}{3}$.

Example 2.—Divide 75 by $3\frac{2}{3}$.

Process

$$3\tfrac{2}{3})\ 75$$
$$11)225$$
$$20\tfrac{5}{11}$$

Explanation.—Since multiplying both dividend and divisor by the same number does not change the quotient,

we can multiply both by the denominator in the divisor, then divide as before.

Example 3.—Divide $125\frac{2}{3}$ by $2\frac{3}{4}$.

Process

$$2\tfrac{3}{4})125\tfrac{2}{3}$$
$$11)502\tfrac{2}{3}$$
$$45\tfrac{2}{3}\tfrac{3}{3}$$

Explanation.—The same as in the preceding, multiply both by the denominator in the divisor. Then divide as in Example 1.

EXERCISES

Divide the following, using the pencil only when necessary:

1. $\frac{5}{7} \div 10$.
2. $\frac{15}{11} \div 5$.
3. $\frac{33}{13} \div 3$.
4. $\frac{8}{13} \div 6$.
5. $\frac{24}{17} \div 8$.
6. $\frac{225}{11} \div 25$.
7. $\frac{169}{15} \div 13$.
8. $\frac{16}{3} \div 32$.
9. $13 \div \frac{1}{3}$.
10. $14 \div \frac{7}{8}$.
11. $\frac{1}{3} \div \frac{4}{3}$.
12. $\frac{5}{8} \div \frac{15}{16}$.
13. $\frac{3}{5} \div \frac{4}{15}$.
14. $\frac{27}{32} \div \frac{3}{1}$.
15. $\frac{18}{5} \div \frac{16}{5}$.
16. $16\frac{1}{2} \div 4$.
17. $362\frac{4}{5} \div 2$.
18. $674\frac{2}{3} \div \frac{1}{3}$.
19. $27\frac{3}{5} \div 9$.
20. $52\frac{1}{4} \div 4$.
21. $84\frac{2}{3} \div 7$.
22. $321\frac{3}{5} \div 3$.
23. $72\frac{5}{8} \div 5$.
24. $46\frac{3}{8} \div 3$.
25. $159\frac{7}{8} \div 4$.

26. If the denominator of a fraction is multiplied by 4, how is the unit of the fraction changed? How is it changed if multiplied by 8? By 7? Illustrate with the fraction $\frac{28}{5}$.

27. If $\frac{1}{10}$ inch on a map represents 40 miles, how many miles are represented by 3 inches on the map?

28. In the blueprint of a house $\frac{1}{4}$ inch in the print represents 1 foot in the actual house. Find the dimensions of the rooms that measure as follows: $2\frac{1}{2}$ by $2\frac{1}{2}$ inches, $4\frac{1}{8}$ by $4\frac{5}{8}$

inches, $5\frac{3}{8}$ by 6 inches, $3\frac{1}{16}$ by $4\frac{1}{32}$ inches, respectively, on the blueprint.

29. A boy wants a pair of ski shoes that costs $\$4\frac{1}{2}$. If he can earn $\$\frac{1}{8}$ per day, how long will he need to work before he can buy the shoes? If one day is 8 hours, how many hours does he work?

30. If a golf ball costs $\$\frac{3}{5}$, how many can be bought for $\$7\frac{1}{5}$?

In the following, x is used for the number that is to be found:

31. $\frac{3}{4} \div 8 = x$.

32. $4 \div x = 8$.

33. $3\frac{1}{7} \div x = 22$.

34. $x \div \frac{2}{5} = 5$.

35. $\frac{6}{13} \div 11 = \frac{x}{13}$.

36. $\frac{6}{9} \div \frac{1}{3} = \frac{x}{9}$.

37. $\frac{7}{5} \div 3 = \frac{7}{x}$.

38. $\frac{19}{3} \div 6 = \frac{19}{x}$.

39. $\frac{3}{5} \div x = \frac{3}{8}$.

40. If a boy can complete $\frac{3}{5}$ of his task in 30 minutes, how long will it take him to do $\frac{1}{5}$ of the task? $\frac{1}{2}$ of the task? $\frac{9}{10}$ of the task?

41. If Tom can build $\frac{1}{3}$ of a boat in 1 day and Dick can build $\frac{1}{4}$ of it in 1 day, what part of the boat can they both build in 1 day? How long will it take both boys working together to build the boat?

42. A boy can mow the lawn in 5 hours, and his brother can mow it in 6 hours. If they borrow another mower and both work together, how long will it take to mow the lawn?

43. On the roof of a factory building is a tank that holds 80 barrels of water and has two pipes opening from it. One of these pipes, when running alone, can empty the tank in 4 hours, and the other, when running alone, can empty it in 16 hours. If both of the pipes are running at the same time, how long will it take to empty the tank? Give two solutions.

44. Pipe number 1 can empty a tank in 4 hr. and pipe number 2 can empty it in 16 hrs. If both pipes run at the same time, how long will it take to empty the tank? See Exercise 43.

45. $\frac{625}{18} \div 175$.

46. $711 \div \frac{33}{10}$.

47. $196 \div \frac{2}{3}$.

48. $300 \div 12\frac{3}{5}$.

49. $5\frac{11}{13} \div 11\frac{5}{13}$.

50. $13\frac{7}{8} \div 1\frac{11}{100}$.

51. $23\frac{4}{5} \div 6\frac{7}{8}$.

52. $5\frac{13}{14} \div 9\frac{26}{28}$.

53. $\frac{1692}{2225} \div \frac{26}{35}$.

54. $20\frac{2211}{1000} \div \frac{1}{100}$.

55. $7\frac{2}{5} \div 3\frac{4}{11}$.

56. $8\frac{4}{33} \div 2\frac{110}{1655}$.

57. $123\frac{3}{10} \div 41\frac{1}{10}$.

58. $910\frac{11}{12} \div 13$.

In the following five exercises, reduce all to fractions, take reciprocals of each divisor, and cancel:

59. $2\frac{1}{3} \times 3\frac{1}{2} \times \frac{4}{17} \times 2\frac{7}{12} \div 20\frac{2}{3}$.

$$2\tfrac{1}{3} \times 3\tfrac{1}{2} \times \tfrac{4}{17} \times 2\tfrac{7}{12} \div 20\tfrac{2}{3} = \frac{7}{3} \times \frac{7}{2} \times \frac{\overset{2}{\cancel{4}}}{17} \times \frac{\overset{31}{\cancel{341}}}{12} \times \frac{\overset{3}{\cancel{6}}}{\underset{2}{\cancel{62}}} = \frac{49}{204}$$

60. $(3\frac{1}{5} \times 5\frac{2}{9}) \div (2\frac{11}{15} \times 10\frac{8}{18})$.

61. $(1\frac{5}{17} \times 22\frac{7}{30} \times \frac{34}{667} \times 11\frac{5}{9}) \div (1\frac{2}{3} \times 4\frac{5}{6} \times 7\frac{8}{9} \div 10\frac{11}{12})$.

62. $(\frac{9}{16} \times \frac{12}{13} \times 14\frac{15}{21} \times 10\frac{26}{31}) \div (\frac{15}{56} \times 10\frac{10}{21} \times \frac{81}{1112})$.

63. $(9\frac{1}{2} \div \frac{19}{26}) \times (12\frac{6}{5} \times 78) \div (2\frac{3}{5} \times \frac{11}{200} \div 1\frac{1}{10})$.

64. Find the values of $\frac{7}{4} \div (\frac{2}{3} + \frac{5}{6}) - \frac{1}{4}$.

Parentheses indicate that the inclosed operations must be performed first. For example, in the above, $\frac{2}{3}$ is added to $\frac{5}{6}$ and then $\frac{7}{4}$ is divided by the sum.

65. Find the value of $3\frac{9}{2} + 7\frac{6}{5} - 1\frac{1}{10} + 80\frac{19}{20}$.

66. Simplify: (*a*) $\frac{5\frac{1}{4}}{7\frac{7}{8}}$. (*b*) $\frac{\frac{17}{50}}{\frac{34}{100}}$. (*c*) $\frac{\frac{2}{3}}{2\frac{2}{3}}$. (*d*) $\frac{5\frac{1}{2}}{\frac{11}{10}}$.

67. Multiply $(5\frac{7}{3} \div 1\frac{1}{12})$ by $\frac{1}{4}$ of $(\frac{3}{4} + \frac{5}{6})$.

68. Find the value of $\dfrac{3\frac{2}{7} \times 8\frac{1}{6}}{4\frac{2}{3} \times 2\frac{1}{16}}$.

Suggestion.—First, multiply $3\frac{2}{7}$ by $8\frac{1}{6}$; second, $4\frac{2}{3}$ by $2\frac{1}{16}$; then divide the first product by the second.

69. $(31\frac{3}{5} - 10\frac{2}{5} + 17\frac{17}{20}) \div (3\frac{7}{10} + 16\frac{1}{2} - \frac{99}{60}) = ?$

70. $(2\frac{2}{3} \div 4\frac{5}{6}) \div (\frac{9}{4} \times \frac{18}{51} \times 2\frac{2}{3}) = ?$

71. Evaluate $\dfrac{\left(2\frac{5}{8} \div \dfrac{4}{\frac{5}{10}}\right) \times 2}{2 - \dfrac{\frac{1}{2}}{4} \div 5}$.

$$\frac{\left(2\frac{5}{8} \div \dfrac{4}{\frac{5}{10}}\right) \times 2}{2 - \dfrac{\frac{1}{2}}{4} \div 5} = \frac{(\frac{21}{8} \div 8) \times 2}{2 - \frac{1}{8} \div 5} = \frac{\frac{21}{8} \times \frac{1}{8} \times 2}{2 - \frac{1}{40}} = \frac{\frac{21}{32}}{\frac{79}{40}} = \frac{21}{32} \times \frac{40}{79} = \frac{105}{316}$$

72. Evaluate $\dfrac{(4\frac{1}{3} - 3\frac{1}{6}) \div 2\frac{3}{4}}{(3\frac{1}{2} + \frac{1}{3}) \div \dfrac{23}{\frac{1}{6}}}$.

73. Add $\frac{3}{4}$ of $\dfrac{5\frac{6}{7}}{\frac{1}{56}}$ to $9 \times (7\frac{1}{3} - 3\frac{2}{9})$.

74. Subtract $\frac{7}{8}$ of $\frac{2}{21}$ from $\frac{11}{12}$ of $\frac{3}{5}$.

75. $\frac{2}{3}$ of 4 is $\frac{5}{6}$ of what number?

76. Find the simplest expression for $\dfrac{22}{3\frac{1}{2}} - \dfrac{3\frac{1}{2}}{2} + \dfrac{9\frac{2}{5}}{1\frac{3}{5}} - 1$.

Perform the operations indicated in the following five exercises:

77. $\dfrac{\frac{2}{3} + \frac{2}{9} + \frac{13}{36} - \frac{11}{12}}{4\frac{1}{4} - 3\frac{1}{3} + 2\frac{1}{2}}$.

78. $\dfrac{(5\frac{1}{6} + 6\frac{1}{5}) \div 2\frac{1}{15}}{1\frac{1}{7} \times 2\frac{2}{7} \times 3\frac{27}{28}}$.

79. $5\frac{1}{5} \times \left(\dfrac{12\frac{3}{4} - 5\frac{11}{12}}{12\frac{11}{12} + \frac{3}{4}}\right) \times \dfrac{1\frac{14}{15}}{1\frac{1}{5} \times \frac{14}{15}}$.

80. $\dfrac{1}{2\frac{3}{4} + 5\frac{6}{7} - 1} \times \dfrac{(2\frac{2}{3} \div 4\frac{4}{6})}{1 \div 18}$.

81. $\left[\dfrac{4\frac{1}{3}}{(5\frac{1}{4} \times 6\frac{2}{3}) + 1}\right] \times (\frac{5}{9} \div 7\frac{2}{3}) \times \left[\dfrac{6\frac{10}{100} \div 10}{(12 - 10\frac{1}{5}) - 1}\right]$.

In the following four exercises, the letters stand for values as follows: $a = \frac{22}{7}$, $b = 14\frac{2}{7}$, $c = 100$, $d = 1\frac{1}{50}$. Find the values of the fractions expressed by the letters.

82. $\dfrac{(a \div b)+c}{c-d}$.

83. $\dfrac{a(b+c)-d}{a(b+c+d)}$.

84. $\dfrac{c-d-(a+b)}{(c \times d)-a}$.

85. $\dfrac{(c \times d) \div (7b)}{(a+b) \div b}$.

86. To put an article on the market, the material costs \$12, the labor costs \$45, the advertising costs \$4, and the transportation costs \$3. What part of the total cost is each separate cost?

87. After driving his car for $22\frac{2}{10}$ months, a man noted that he had gone $19{,}264\frac{2}{10}$ miles. What was the average number of miles driven per month?

88. How many articles costing $\$\frac{3}{5}$ each can be bought for $\$16\frac{1}{5}$?

89. The height of a book case inside is 6 feet $6\frac{1}{8}$ inches. There are to be inserted six shelves each $\frac{7}{8}$ inch thick and equally spaced. Find the clearance between the shelves.

90. A man drove his car 415 miles in 10 hours 15 minutes. How far can he drive at the same rate in $18\frac{1}{6}$ hours?

91. A carpenter is to cut nine shelves each 2 feet 8 inches long. There are boards 10, 12, 14, and 16 feet long from which to cut the shelves. Which boards and how many should be chosen so as to cut with the least waste?

92. Two places A and B are 24 miles apart on a river that flows 3 miles an hour. A man can row 5 miles an hour in still water. He goes from A to B and back. Find the time for the journey.

93. If a man can do a piece of work in 10 hours, what part of it can he do in 1 hour? in 2 hours? in 5 hours? in $\frac{1}{2}$ hour?

94. If a man can do $\frac{1}{4}$ of a piece of work in 1 hour, in how many hours can he do all the work? $\frac{3}{4}$ of the work?

95. If a man can do $\frac{5}{7}$ of a piece of work in 5 hours, in how many hours can he do all the work? $\frac{2}{7}$ of the work? $\frac{1}{2}$ of the work? $\frac{1}{49}$ of the work?

96. Three pipes can empty a reservoir in 6, 5, and 4 hours respectively. How long will it take them to empty it if running together?

Solution.—Since the first pipe can empty the reservoir in 6 hours, it can empty $\frac{1}{6}$ of the reservoir in 1 hour. Reasoning in the same manner, the second pipe can empty $\frac{1}{5}$ of it in 1 hour, and the third pipe $\frac{1}{4}$ of it in 1 hour. Hence, the three pipes can empty $\frac{1}{6}+\frac{1}{5}+\frac{1}{4}=\frac{37}{60}$ of the reservoir in 1 hour. To empty the whole reservoir, it will take all the pipes together, $1\div\frac{37}{60}=1\frac{23}{37}$ hours.

97. In the preceding exercise, how long will it take to empty the reservoir if it is full to begin with, and the first two pipes are emptying out of, and the third emptying into, the reservoir?

98. Pipes numbered 1, 2, and 3 open from a tank. Pipe 1 can empty the tank in 8 hours; pipe 2 can empty it in 6 hours; and pipe 3 can empty it in 2 hours. How many hours will it take to empty the tank if all three of the pipes are running at the same time?

99. In the previous exercise the tank is full to begin with and the first and third pipes are emptying out of, and the second emptying into the tank. How long will it take to empty the tank?

100. A man can paint his garage in 8 hours, and his son can paint it in 12 hours. How long will it take to paint the garage if father and son work together? What part can each do in 1 hour? What part can both working together do in 1 hour?

101. A father earns enough money in $60\frac{1}{2}$ days to buy a car, and his son earns enough in $88\frac{1}{2}$ days to buy the car. What part of the cost of the car can they earn in 1 day if they work together? If they work together, how long will it take them to earn enough to buy the car?

102. The plant property of a company is as follows: land \$940,000; building \$7,500,000; machinery, tools, and dies \$16,200,000; office equipment \$50,000. What part of the plant property is the land? What part is the building? What part is machinery, tools, and dies? What part is office equipment? If the total assets of the company are \$33,800,000, what part of this total assets is each of the above?

103. A contractor is to build a house in 40 days. If 16 men work on the house for 30 days and do $\frac{1}{2}$ of the estimated work, how many men must work for the next 10 days to finish the house on time?

104. A contractor agrees to do some grading in 15 days. If 10 men work 12 days and do only $\frac{2}{3}$ of the grading, how many men must work the remaining 3 days to finish the work?

105. A piece of work when forged weighed $214\frac{1}{2}$ pounds. After being turned down, it weighed $156\frac{3}{4}$ pounds. The forging cost $16\frac{1}{2}$ cents per pound, and the metal turned off sold at $3\frac{1}{4}$ cents per pound. Find the net cost of the metal in the finished piece.

106. The following rule is often used to find the weight of round steel and wrought iron: Square the diameter in inches and multiply by $\frac{8}{3}$; the product is the weight in pounds of 1 foot of the bar. (The product of a number multiplied by itself is the square of the number.) Using this rule, find the weights of round bars of steel of the following dimensions:

(1) Diameter $\frac{5}{8}$ inch, length $10\frac{1}{2}$ feet.

(2) Diameter $2\frac{3}{4}$ inches, length $2\frac{1}{8}$ feet.

(3) Diameter $6\frac{1}{2}$ inches, length $1\frac{5}{8}$ feet.

107. The distance F, in Fig. 5, across the flats in a bolthead or nut, either a square or a hexagon, is equal to $1\frac{1}{2}$ times the diameter of the bolt plus $\frac{1}{8}$ inch. As a formula this is

$$F = 1\tfrac{1}{2}D + \tfrac{1}{8}.$$

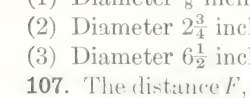

FIG. 5

Test the widths across the flats in the following table taken from a manufacturer's catalogue:

Diameter of bolt D	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	$1\frac{1}{8}$	$1\frac{5}{8}$	$1\frac{3}{4}$	2
Width across flats F	$\frac{1}{2}$	$\frac{19}{32}$	$\frac{11}{16}$	$\frac{25}{32}$	$1\frac{1}{16}$	$1\frac{1}{4}$	$1\frac{7}{16}$	$1\frac{13}{16}$	$2\frac{9}{16}$	$2\frac{3}{4}$	$3\frac{1}{8}$

108. To change from centigrade thermometer reading to Fahrenheit, the following formula is used: $F = \frac{9}{5}C + 32$ degrees, where C is the centigrade reading and F the Fahrenheit.

(*a*) Find F when $C = 100°$.

(*b*) Find F when $C = 950\frac{5}{9}°$.

109. To change a Fahrenheit thermometer reading to centigrade, the following formula is used: $C = \frac{5}{9}(F - 32°)$, where C is the centigrade reading and F the Fahrenheit.

(*a*) Find C when $F = 81\frac{1}{2}°$.

(*b*) Find C when $F = 1760°$.

110. In inspecting steam boilers, the following formula is often used:

$$t = \frac{PRF}{T \times \%}$$

where

t = thickness of plate in inches.

P = steam pressure in pounds per square inch.

R = radius of boiler in inches ($\frac{1}{2}$ of diameter).

F = factor of safety.

T = tensile strength of boiler plate in pounds.

$\%$ = percentage of strength in joints.

Find the thickness of the boiler plate which should be used for a boiler 50 inches in diameter to carry 120 pounds of pressure if the tensile strength is 60,000 pounds. Use 50% as the strength of the joints and a factor of safety of 6.

111. Find the thickness of the boiler plate for a 72-inch boiler to carry a pressure of 90 pounds with a tensile strength of 50,000 pounds. Use 50% as the joint strength and a factor of safety of 6.

112. The following formula is used in finding the diameter of a steam boiler

$$D = \frac{2tT \times \%}{PF},$$

where

D = diameter of boiler in inches.

t = thickness of plate in inches.

T = tensile strength of boiler plate in pounds.
P = steam pressure in pounds per square inch.
F = factor of safety.
% = percentage of strength in joints.

Find the diameter for a steam boiler having a $\frac{5}{8}$-inch plate, allowing 50% for strength of joints and a factor of safety of 6, with a tensile strength of 60,000 pounds, and 125 pounds pressure per square inch.

SCREWS AND SCREW GEARING

29. Screws and screw threads.—Screws are commonly used for fastening together wooden or metal parts. For convenience in entering wood, a wood screw is tapered while a machine screw is the same diameter throughout and is often fitted with a nut.

The thread of a screw is formed by cutting a spiral groove around the body of the screw. Screw threads used for fastening are made by means of dies and taps. A die is a tool used to make external threads on the body of a screw or bolt, and a tap is a tool for making an internal thread as in a nut.

The threads on parts used in machines and for communicating motion are made by a cutting process by using a lathe and a single-pointed cutting tool of the proper size. The grooves and threads formed are of various sizes and several shapes depending on the size of the screw and the purposes for which the screw, bolt, or other device is to be used. The different kinds of threads will be studied later (see Part III, Art. 35).

30. Pitch and lead of screw threads.—The **pitch** of a screw thread is the distance from, say, the center of the top of one thread to the center of the top of the next.

The **lead** of a screw thread is the distance the screw will move forward in a nut for each complete turn of the screw.

In a **single-threaded screw** there is only one groove cut spirally any number of times around the body of the screw; in a **double-threaded screw** there are two grooves running side by side spirally around the screw;

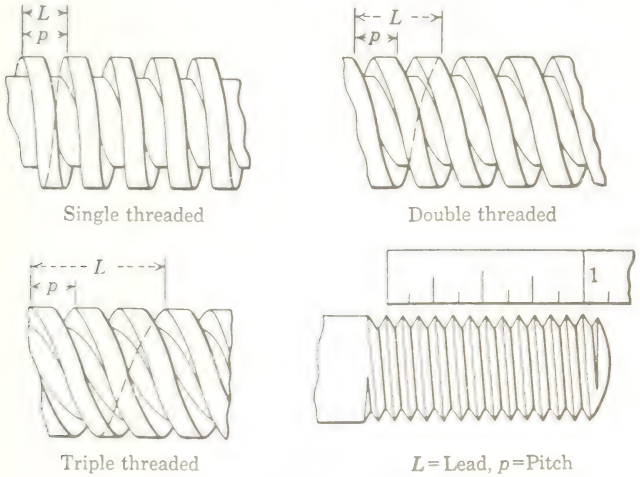

FIG. 6.

and a **triple-threaded screw** has three grooves running around the screw.

For a single-threaded screw the pitch and the lead are equal; but for a double-threaded screw, the lead is twice the pitch; and for a triple-threaded screw, the lead is three times the pitch.

The meaning and relations of **pitch** and **lead** will be made clear by reference to Fig. 6. The pitch can be easily determined by placing a scale on the screw as

shown in Fig. 6 and counting the number of threads for one inch. If the screw is single-threaded, this is the number of times the screw must be turned around to advance 1 inch. This is often called the *pitch*. Thus, a 10-pitch single-threaded screw requires 10 turns to advance 1 inch.

31. Screw gearing.—**Spiral** or **screw gearing** is often used where it is desired to reduce the speed greatly.

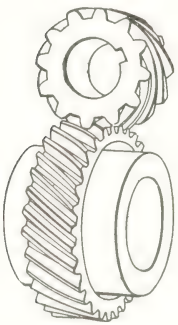

FIG. 7.

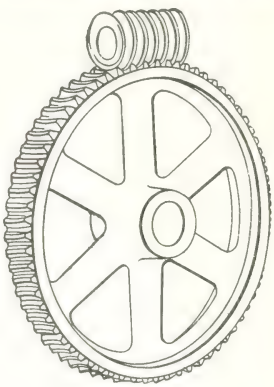

FIG. 8.

The teeth are arranged in the same manner as the threads of a screw. A screw wheel may have one tooth or any number of teeth. A one-toothed wheel corresponds to a one-threaded screw, a many-toothed wheel to a many-threaded screw.

In Fig. 7, the upper wheel has 12 teeth, and corresponds to a 12-threaded screw; whereas the lower wheel has 36 teeth and corresponds to a 36-threaded screw. Here the smaller wheel makes three revolutions while the larger is making one. If the smaller had but one

tooth or was single-threaded, it would make 36 turns to one turn of the larger wheel.

When the number of threads or teeth on the smaller wheel is few, the smaller wheel is called a **worm,** and the larger the **worm wheel.** Figure 8 shows a worm and a worm wheel. The worm is single-threaded with a left-handed thread.

EXERCISES

1. What is the lead of a single-threaded screw that advances $\frac{1}{2}$ inch in eight turns? What is the pitch?

2. What is the lead in a double-threaded screw that has 18 threads to an inch? What is the pitch?

3. A jackscrew is single-threaded and has three threads to an inch. How far does it advance in $\frac{1}{3}$ of a turn? How many turns will advance it 6 inches?

4. A single-threaded screw with 36 threads to an inch will advance how far in $\frac{1}{10}$ of a turn? In $1\frac{1}{5}$ turns? In $16\frac{1}{2}$ turns? In how many turns will it advance $\frac{11}{20}$ inch?

5. According to the Franklin Institute standards for the dimensions of bolts and nuts, a $\frac{3}{8}$-inch bolt has 16 threads per inch. What is the pitch? The lead, if single-threaded? How many full turns of the nut will it take to advance the bolt $1\frac{3}{4}$ inches?

6. A $4\frac{3}{4}$-inch bolt has $2\frac{5}{8}$ threads per inch. What is the pitch? What is the lead if triple-threaded?

7. In a special threaded screw for a screw-power stump puller, the screw is double-threaded with a pitch of $\frac{11}{16}$ inch. How many turns of the nut are required to lift a stump $3\frac{1}{4}$ feet?

8. If a worm is 2-pitch and single-threaded, how many inches will it cause the circumference of the worm wheel to advance for one revolution of the worm? How many inches if double-threaded? If triple-threaded? If 6-threaded? (2-pitch means 2 threads to the inch.)

9. If a single-threaded worm, making 20 revolutions per second, turns a worm wheel having 54 teeth, Fig. 8, how many revolutions per minute (R. P. M.) will the worm wheel make?

10. How many turns must be made with a triple-threaded screw having $6\frac{1}{4}$ threads to the inch to have it advance a distance of $2\frac{5}{8}$ inches?

11. The lead screw on the table of a milling machine has a double thread with a pitch of $\frac{1}{4}$ inch. How many inches per minute is the feed if the lead screw is making 2 R. P. M.?

12. In turning a piece in a turning lathe the distance the cutting tool advances along the piece for each revolution is called the *feed*. The feed is usually given as a fraction of an inch. How long will it take to turn a piece $2\frac{1}{2}$ feet long, if it makes 17 R. P. M., and the feed is $\frac{1}{16}$ inch?

13. How many turns per minute is a piece making if a length of $2\frac{3}{4}$ feet is turned in 80 minutes when the feed is $\frac{1}{16}$ inch?

14. What feed is necessary to run a cut 50 inches in 8 minutes at 40 R. P. M.?

15. In planing an aluminum casting of width $12\frac{1}{2}$ inches and length $15\frac{1}{4}$ inches, find the time required if cutting speed is 40 feet per minute, return speed 80 feet per minute, and feed $\frac{1}{32}$ inch.

16. A planer is belted so as to give a cutting speed of 40 feet per minute and a return speed twice as fast. Find the time for one stroke and return on a piece 4 feet 10 inches long if the overrun at each end to clear the tool is 1 inch.

17. How long will it take to make 44 cuts on a casting 2 feet 1 inch long at 45 feet per minute if the overrun is 1 inch at each end, and the return is 2 to 1?

18. Find the time for planing a casting 2 feet 9 inches long and 1 foot 6 inches wide if cutting speed is 96 feet per minute, return speed 2 to 1, overrun 1 inch at each end, and the feed $\frac{3}{16}$ inch.

19. On a casting 1 foot 11 inches long a 3 to 1 planer is making 18 strokes a minute with 1 inch overrun at each end. What is the cutting speed?

20. A $\frac{7}{16}$-inch twist drill (Fig. 9) has a speed of 130 R. P. M. when cutting steel. How long will it take to drill through a $\frac{5}{8}$-inch plate if 120 revolutions are required to drill 1 inch?

Fig. 9.

21. A $1\frac{3}{16}$-inch drill makes 66 R. P. M. in iron and has a feed of $\frac{1}{8}\frac{1}{6}$ inch. How long will it take to drill 20 holes through a $\frac{3}{4}$-inch plate if $\frac{1}{2}$ minute is allowed for setting for each hole?

22. In drilling through mild steel $1\frac{5}{8}$ inches thick, a hole 1 inch in diameter is drilled in $1\frac{3}{4}$ minutes. Find the distance drilled per minute.

23. A $\frac{5}{16}$-inch drill can make 320 R. P. M. in brass. If 120 turns are made to drill 1 inch, find how many holes can be drilled in a $\frac{5}{8}$-inch plate in 1 hour if $\frac{1}{4}$ the time is used in setting the drill.

24. A machinist drills six holes through a piece that is $2\frac{3}{4}$ inches thick. The drill is $1\frac{7}{8}$ inches in diameter and makes 154 R. P. M. with a feed of $5\frac{1}{6}$ inch. How many minutes does it take if 3 minutes are used in setting for each hole?

25. A hole $6\frac{3}{4}$ inches deep is drilled with a $1\frac{1}{8}$-inch drill making 126 R. P. M. What feed is required to drill the hole in $3\frac{3}{4}$ minutes?

CHAPTER III

DECIMAL FRACTIONS

32. Fractions that have 10, 100, 1000, etc., for denominators are **decimal fractions.**

Thus, $\frac{53}{100}$, $\frac{3756}{10000}$, $\frac{76}{1000}$, $\frac{4326}{1000}$ are decimal fractions.

In writing a decimal fraction it is convenient to omit the denominator, and indicate what it is by placing a point (.), called a **decimal point,** in the numerator so that there shall be as many figures to the right of this point as there are zeros in the denominator.

Thus, $\frac{53}{100}$ is written .53; $\frac{3756}{10000} = .3756$; $\frac{76}{1000} = .076$; $\frac{4326}{1000} = 4.326$.

Often, in writing decimals, a zero is placed at the left of the decimal point when there is no whole number figure. Thus, .53 is written 0.53. The best practice uses the zero in this way before the decimal point in printed books.

It is to be noted that when there are fewer figures in the numerator than there are zeros in the denominator, zeros are added on the left of the figures to make the required number.

From the meaning of the decimal fraction, it is seen that the misplacing of the decimal point changes the meaning greatly. For each place it is moved to the right, the value of the decimal fraction is multiplied

by 10; and for each place it is moved to the left, the value is divided by 10.

Thus, 2.75 becomes 27.5 when the point is moved one place to the right and 0.275 when the point is moved one place to the left. In the first case 2.75 is multiplied by 10, and in the second case it is divided by 10.

It is well to recall the fact that when we have a number such as 3333, where the same figure is used throughout, the values expressed by the threes vary greatly. For every place a three is moved toward the left, its value is increased ten times; and, as we pass from left toward the right, each three has one-tenth the value of the one to the left of it. Since the above relations hold when we pass to the right of the place representing units, we have the following relative values of the places:

Thousands	Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths	Ten-millionths	Hundred-millionths
0	0	0	0	.	0	0	0	0	0	0	0	0

33. Historical note.—Decimal fractions were invented by **Simon Stevin** of Belgium who lived from 1548 to 1620. He first described them in his *La Disme*, published in 1585. He did not use the decimal point but made use of a rather cumbersome notation. The name comes from the Latin *decem* meaning *ten*. The decimal point

was first used by *Pitiscus* in 1612. At the present time the decimal point is printed at the center of the figures in books printed in England, as 2·53; and the decimal comma is used instead of the decimal point in German books, as 2,53.

The invention of decimal fractions is considered one of the great inventions for shortening modern calculations. One would suppose that the invention of decimal fractions would be the natural outgrowth of the Hindu system of writing numbers; but it was more than a thousand years after the Hindu system had been brought to perfection that decimal fractions were invented, and it was more than a hundred years later that decimal fractions came into common use in arithmetics.

34. Reading numbers.—The whole number 23,676 is read *twenty-three thousand six hundred seventy-six*. It should be noticed that no word "*and*" is used in reading a whole number.

A decimal is read like a whole number except that the name of the right-hand place is added.

For example, the number 0.7657 is read *seven thousand six hundred fifty-seven ten-thousandths.*

When a whole number and a decimal fraction are written together the word *and* is used between the two parts in reading.

Thus, 73.2658 is read seventy-three *and* two thousand six hundred fifty-eight ten-thousandths.

Where one person is reading numbers for another to write, it is not customary to proceed in the above manner.

Thus, the number 23.6785 may be read *twenty-three, point, sixty-seven, eighty-five*. Or we may read it, *two, three, point, six, seven, eight, five.*

EXERCISES

Write the following in figures:

1. Three hundred fifty-six and ten thousandths.
2. Two hundred fifty-six and twenty-three thousandths.
3. One hundred fifty-five millionths.
4. Four hundred fifty-six thousandths.
5. Four hundred and fifty-six thousandths.
6. Three hundred twenty-five and twenty-five ten-thousandths.

Read the following:

7. 23.462.
8. 2003.203.
9. 0.4256.
10. 4200.0056.
11. 1200.3604.
12. 10,101.2301.
13. 5867.0067.
14. 10,000.0001.

35. Reduction of a common fraction to a decimal fraction.—A decimal fraction differs from a common fraction only in having 1 with a certain number of zeros annexed for the denominator. The common fraction can then be changed to a decimal by reducing it to a fraction having 1 with zeros for a denominator.

It is evident from the method of reducing a common fraction to one with a different denominator that a common fraction can be changed to a decimal only when its denominator is contained an exact number of times in 10, 100, 1000, or 10,000, etc.

Thus, $\frac{2}{5} = \frac{4}{10}$ or 0.4, and $\frac{9}{16} = \frac{5625}{10000}$ or 0.5625; but $\frac{2}{7}$ cannot be expressed exactly as a decimal because 7 is not exactly contained in 10, 100, or 1000, etc.

To reduce a common fraction to a decimal proceed as follows:

RULE.—*Annex zeros to the numerator and divide by the denominator; place the decimal point so as to make as many decimal places in the result as there were zeros annexed.*

Thus, $\frac{7}{8} = 0.875$.

Process

$$8)\underline{7.000}$$
$$0.875$$

And $\frac{2}{7} = 0.2857+$.

Process

$$7)\underline{2.0000}$$
$$0.2857+$$

The sign + placed after the number indicates that there are still other figures if the division is carried further.

A common fraction in its lowest terms will reduce to an exact decimal only when its denominator contains no other prime factors than 2 and 5.

Thus, $\frac{3}{64}$ reduces to an exact decimal for 64 is made up of $2\times2\times2\times2\times2\times2$, while $\frac{7}{12}$ cannot be reduced to an exact decimal for its denominator contains the factor 3.

36. Decimal fraction to common fraction.—To change a decimal fraction to a common fraction:

RULE.—*Replace the decimal point by a denominator having* 1 *and as many zeros as there are decimal places in the original fraction.* (See Art. 32.)

Thus, $2.375 = \frac{2375}{1000}$, which may be written as a mixed number, $2\frac{375}{1000} = 2\frac{3}{8}$.

EXERCISES

Reduce the following to decimals:

1. $\frac{7}{8}$.
2. $\frac{5}{3}$.
3. $\frac{19}{20}$.
4. $\frac{762}{25}$.
5. $11\frac{11}{32}$.
6. $505\frac{606}{2000}$.

Reduce the following to common fractions or mixed numbers in their lowest terms:

7. 0.4040.
8. 0.72.
9. 0.00675.
10. 0.0101.
11. 15.0375.
12. 510.0051.

13. Reduce the following decimals of an inch to common fractions in their lowest terms: 0.625; 0.875; 0.734375; 0.00125; 0.453125.

14. Express the following in their simplest common fractional form: $6.6\frac{2}{3}$; $0.0100\frac{1}{2}$; $0.87\frac{1}{2}$; $0.0083\frac{1}{3}$; $0.0163\frac{1}{2}$; $0.14\frac{2}{7}$; $0.9\frac{1}{11}$.

15. Change the following per cents to their simplest common fractional forms: $62\frac{1}{2}\%$; $166\frac{2}{3}\%$; $12\frac{1}{2}\%$; $\frac{3}{4}\%$; $71\frac{3}{7}\%$; $212.2\frac{5}{8}\%$.

16. Tell without trial which of the following common fractions will reduce to exact decimals: $\frac{11}{16}$; $\frac{2}{5}$; $\frac{2}{7}$; $\frac{17}{10}$; $\frac{17}{45}$; $\frac{101}{625}$; $\frac{9}{10}$; $\frac{55}{62}$; $\frac{23}{24}$; $\frac{123}{640}$; $\frac{99}{176}$.

17. Change the following decimals of an inch to the nearest 64ths of an inch: 0.1563; 0.1094; 15.63; 1.38; 0.078.

37. Addition of decimals.

RULE.—*Write the numbers so that the decimal points are under each other. Add as in whole numbers, and place the decimal point in the sum under the other decimal points.*

Example.—Add 36.036, 7.004, 0.00236, 427, 723.0026.

$$\begin{array}{r} 36.036 \\ 7.004 \\ 0.00236 \\ 427. \\ 723.0026 \\ \hline 1193.04496 \end{array}$$

38. Subtraction of decimals.

RULE.—*Write the numbers so that the decimal points are under each other. Subtract as in whole numbers, and place the decimal point of the remainder under the other decimal points.*

Example.—Subtract 46.8324 from 437.421

$$\begin{array}{r} 437.421 \\ 46.8324 \\ \hline 390.5886 \end{array}$$

39. Multiplication of decimals.

RULE.—*Multiply as in whole numbers, and point off as many decimal places in the product as the sum of the numbers of the places in the factors.*

Example 1.

Multiply 7.32 by 0.032.

$$\begin{array}{r} 7.32 \\ 0.032 \\ \hline 1464 \\ 2196 \\ \hline 0.23424 \end{array}$$

Example 2.

Multiply 0.00264 by 0.000314.

$$\begin{array}{r} 0.00264 \\ 0.000314 \\ \hline 1056 \\ 264 \\ 792 \\ \hline 0.00000082896 \end{array}$$

Multiplying a whole number or a decimal by 0.1 moves the decimal point one place to the left; by 0.01, two places; by 0.001, three places; etc. If it is necessary, zeros are prefixed to the multiplicand.

Thus, $32.4 \times 0.0001 = 0.00324$.

Multiplying by 10, 100, 1000, etc., moves the decimal point 1, 2, 3, etc., places to the right.

40. Division of decimals.

RULE.—*If the number of decimal places in the dividend is less than the number in the divisor, annex zeros to the dividend till there are as many as or more decimal places than in the divisor. Divide as in whole numbers, and point off as many decimal places in the quotient as the number of decimal places in the dividend exceeds the number in the divisor.*

Example 1.

Divide 0.4375 by 0.125.

$$\begin{array}{r|l} 0.4375 & 0.125 \\ \hline 375 & 3.5 \\ \hline 625 & \\ 625 & \\ \hline \end{array}$$

Example 2.

Divide 4365 by 0.005.

$$\begin{array}{r} 0.005)4365.000 \\ \hline 873000 \end{array}$$

Dividing by 0.1, 0.01, 0.001, etc., moves the decimal point 1, 2, 3, etc., places to the right. Dividing by 10, 100, 1000, etc., moves the decimal point 1, 2, 3, etc., places to the left.

41. Accuracy of results.— Often we are asked to give a result correct to a certain number of decimal places.

Thus, if in working a problem we have a result as 47.264735, and wish to write it correct to three places, it is 47.265−; correct to two places, it is 47.26+; correct to one place, 47.3−; correct to five places, 47.26474−.

The last place taken is written one larger when the next figure to the right is 5 or more.

The part to the right of the last place taken is thrown away when the first figure of it is less than 5.

In this way we call a half or more of the last unit taken, a whole one of those units, and throw away anything less than a half.

The + sign is used to show that the accurate result is larger than the one given, that is, that something has been thrown away; and the − sign is used to show that the accurate result is smaller than the one given, that is, that something has been added.

EXERCISES

Add up and test by adding down:

1.

648.03
17.895
219.921
97005.007
9.098
100.001
5678.91

2.

897.1
0.989
900.76
91901.359
9876.
101.109
77.007

3.

92.928
94.987
60.768
19.657
43.542
76.305
58.143

4. $13\frac{1}{4}+14\frac{1}{3}+77\frac{1}{2}+12.5+28.675+15\frac{2}{3}+\frac{1}{2}=?$

5. $11\frac{1}{9}+66\frac{2}{3}+1\frac{2}{9}+125.125+375.375+10\frac{1}{2}=?$

6. $78.808+202.202+62\frac{1}{2}+98\frac{3}{20}+10\frac{3}{4}+111.1=?$

7. $1-0.69897=?$

8. $2-1.30103=?$

9. $4.641588-4.626=?$

10. $3.1416-1.4142=?$

11. $1.73205-1.44225=?$

12. $75.7575-55.1\frac{1}{5}=?$

13. From one hundred take seven thousandths.

14. From nine million nine take nine hundred and nine thousandths.

15. From one hundred and one-tenth take one and ten thousandths.

16. One quart liquid measure has 57.75 cubic inches, and 1 quart dry measure has 67.200625 cubic inches. How many cubic inches larger is the dry quart than the liquid quart?

17. $3.62 \times 0.0037 = ?$ **20.** $7.789 \times 4.924 = ?$

18. $2.53 \times 0.00635 = ?$ **21.** $2.236 \times 799 = ?$

19. $0.00076 \times 0.0015 = ?$ **22.** $2.967 \times 2.967 = ?$

23. $8.943 \times 1\frac{2}{3} = ?$

Process

$$
\begin{array}{rl}
8.943 & \\
\quad 1\frac{2}{3} & \\
\hline
2\ 981 & = \frac{1}{3} \text{ of } 8943 \\
5\ 962 & = \frac{2}{3} \text{ of } 8943 \\
8\ 943 & = 1 \times 8943 \\
\hline
14.905 & = 1\frac{2}{3} \times 8.943
\end{array}
$$

Why would it be best not to reduce $1\frac{2}{3}$ to a decimal before multiplying? The multiplication can be carried out readily as shown here. The process is as if the multiplicand were a whole number.

24. $2.55 \times 4\frac{3}{5} = ?$ **26.** $1\frac{2}{5} \times 1.4142 \times 61 \times 6.5 = ?$

25. $0.0506 \times 10\frac{1}{2} = ?$ **27.** $8.4 \times 0.0105 \times 1.0055 = ?$

28. $9876.5 \times 0.0011 \times 0.091 = ?$

29. $0.6\frac{5}{8} \times 8\frac{5}{8} \times 6.6705 = ?$

30. Multiply $5\frac{3}{4}$ thousandths by $5\frac{3}{4}$ hundredths.

31. One kilogram equals 2.2046 pounds. How many pounds in 106.5 kilograms?

32. Multiply each of the following numbers by 0.1, 0.01, 0.001, 0.0001, 10, 100, 1000, 10,000:

86; 78.125; 0.0021; 25.25.

33. $33(6.25) \div 8.25 = ?$ **34.** $90.58 \times 2.2046 = ?$

35. Divide 43.769 by 4.76 correct to four decimal places.

Process

$$
\begin{array}{r|l}
43.769000 & 4.76 \\
4284 & 9.1951 \\
\hline
929 & \\
476 & \\
\hline
4530 & \\
4284 & \\
\hline
2460 & \\
2380 & \\
\hline
800 & \\
476 & \\
\hline
324 &
\end{array}
$$

Explanation.—Since the quotient is to be correct to four places, the dividend must contain four more decimal places than the divisor. Three zeros are added to make this number. Since the fifth decimal figure in the quotient is not less than 5, the answer is $9.1952-$.

In the next five exercises, find the result correct to four decimal places

36. $9.375 \div 4.76 = ?$

37. $89.7201 \div 3.276 = ?$

38. $34.675 \div 4.375 = ?$

39. $43.45 \div 3.1416 = ?$

40. $3.1416 \div 6.67 = ?$

41. Divide 324.8 by 4000.

Process

$$
\begin{array}{r|l}
4000 & 0.3248 \\
\hline
 & 0.0812
\end{array}
$$

Explanation.—Cancel the zeros in the divisor. Since this divides the divisor by 1000, the dividend must be divided by 1000, this is done by moving the point three places to the left.

Perform the following divisions without a pencil:

42. $5 \div 200$; $2 \div 500$; $0.0099 \div 11{,}000$; $1.69 \div 1300$; $2.075 \div 5000$; $2828 \div 0.014$.

43. $0.1 \div 0.00001$; $0.99 \div 0.0033$; $0.00064 \div 1.6$; $0.096 \div 12{,}000$.

44. Divide $3.1416 \times 1.25 \times 50$ by $0.8 \times 2.75 \times 3$.

Process

$$\frac{\overset{119}{\cancel{3.1416}\times\cancel{1.25}\times\cancel{50}}}{\underset{11}{\cancel{0.8}\times\cancel{2.75}\times\cancel{3}}} = 29.750.$$

Explanation.—The canceling may be done as in whole numbers, paying no attention to the decimal point. When through, point off as many places in the result as the difference between the sum of those above and the sum of those below the line. Thus, in the example there are six places above and three below the line; hence, the result has three decimal places.

Find the value of the following:

45. $\dfrac{37.5 \times 60.6 \times 200}{2.5 \times 303 \times 0.2}$ to units.

46. $\dfrac{3.1416 \times 2.2 \times 25 \times 88}{1.25 \times 0.11 \times 40}$ to three places.

47. $\dfrac{8.2 \times 2.5 \times 10.8 \times 0.96}{41 \times 200 \times 1.2}$ to four places.

48. $\dfrac{0.7854 \times 60 \times 12.5 \times 5280}{231 \times 0.025 \times 300}$ to two places.

49. $\dfrac{5.8 \times 8.25 \times 10.1 \times 1.732}{60.60 \times 0.25}$ to two places.

50. For the fiscal year ending Jan. 31, 1939, the consolidated net income of a company was $23,354,363. If the company has 5,588,030 shares of capital stock, what was the net earned per share?

51. A company lost $3.375 per share on 265,880 shares of stock. What was the total loss?

52. If carpet is quoted at \$4.875 per square yard, what would a rug of 12 square yards cost?

53. A company hires 12 men for 87.5 cents per hour and 9 girls at 62.5 cents per hour. What would be the weekly pay roll if they all work $6\frac{1}{2}$ days of 8 hours each?

54. What is the inside diameter of a pipe which is 6.84 inches outside diameter and made of iron $\frac{3}{16}$ inch thick?

55. A cubic foot of brass is drawn in the form of a circular cylinder of diameter 0.0625 inch. How many miles long is the wire? [The volume of a cylinder is 3.1416 × (the square of its radius) × (its length); 1 mile is 5280 feet.]

56. A pump delivers 16.16 gallons per stroke. What weight of water will it deliver in 120 strokes? (One gallon of water weighs 8.355 pounds.)

57. In 1 pound of phosphor bronze 0.925 is copper, 0.07 is tin, and 0.005 is phosphorus. How much of each is there in $25\frac{1}{8}$ pounds of phosphor bronze?

58. Add $\frac{5\frac{1}{5}}{7\frac{1}{5}}$ and $\frac{4\frac{1}{4}}{8\frac{1}{8}}$, divide the result by $(\frac{8}{65} \div \frac{9}{10})$, and change the result to a decimal.

59. From $\frac{2}{7} \times 1.732$ subtract the product of 0.4343 and $3\frac{1}{3}$, divide the remainder by $\frac{2}{2}\frac{1}{5}$, and change the result of a decimal.

60. Simplify $\dfrac{(\frac{1}{2} \div 3\frac{1}{4}) \times 0.00025}{0.075}$.

61. Simplify $\dfrac{(3.71 - 1.908) \times 7.03}{2.2 - \frac{7}{3}\frac{4}{3}\frac{4}{3}}$.

62. Simplify $\dfrac{(201 + 2.25 \times 0.004) \div (1.0337 - 31.09 \times 0.03)}{4.5 \div 960}$.

63. A child paid 15 cents for $\frac{1}{4}$ pound of cookies. The cookies were equally mixed from one box marked 32 cents per pound and another box marked 28 cents per pound. Explain how the error was made.

64. The length of heat waves varies from 0.810 micron to 314.00 microns. The longest heat wave is how many times as long as the shortest heat wave?

65. A car is going 1.125 miles per hour. How long will it take this car to go $468\frac{3}{4}$ miles?

66. A layer of No. 8 wire, which is 0.162 inch in diameter, is wound on a pipe $24\frac{3}{8}$ inches long. How many turns of wire are wound on the pipe?

67. What will be the cost of 7.2 miles of railroad track if the rails weigh $42\frac{2}{3}$ pounds per foot and cost \$41.75 a ton of 2000 pounds? (1 mile = 5280 feet.)

68. An iron bar is 10.18 inches long, 3.45 inches wide, and 0.87 inch thick. Find its weight if 1 cubic inch of iron weighs 0.28 pound.

69. Find the value of the following building material: 27,750 shingles at \$4.25 per thousand; 47,256 feet of lumber at \$45 per thousand; 126,450 bricks at \$7.75 per thousand.

70. It has been determined by experiment that each square foot of steam radiation will give off to the surrounding air about 3 heat units per hour per degree difference between the air in the room and the steam radiator. If the temperature of the radiator is 212 degrees and that of the room 70 degrees, how many heat units will be given off per hour on 24,000 square feet of radiating surface? How many pounds of coal will it take to make this steam if 1 pound of coal contains 10,000 heat units?

71. Nickel steel will stand a pull of about 90,000 pounds per square inch in cross section. What pull will a bar 1.125 inches wide and 0.875 inch thick stand?

72. The composition of white metal as used in the Navy Department is as follows: tin 7.6 parts, copper 2.3 parts, zinc 83.3 parts, antimony 3.8 parts, and lead 30 parts. Find the number of pounds of each in 1270 pounds of white metal.

73. The operating budget for Chicago Girl Scouts for 1939 was \$40,000. This amount was financed as follows: contributions by friends $\frac{9}{20}$, annual cooky sales $\frac{5}{20}$, scout shops $\frac{3}{20}$, community fund $\frac{3}{20}$. Find the amount of each of the four different financing methods.

74. A reamer that is 6 inches long is 1.2755 inches in diameter at the small end and 1.4375 inches at the larger end.

Find the taper per foot. (The taper per foot means the decrease in diameter per foot of length.)

75. A man has $256 in a closed bank. At different times he was paid $\frac{1}{4}$, $\frac{1}{5}$, $\frac{3}{20}$, $\frac{1}{10}$, and $\frac{1}{20}$ of the total amount. How much was he paid, and how much remains to be paid?

76. For the year 1938 the United States railroads found that 1 pound of coal would pull 8.8 tons of freight 1 mile, and that it took 14.7 pounds of coal to haul 1 passenger-train car 1 mile. At this rate, how many pounds of coal would it take to send an automobile by freight a distance of 425 miles if the automobile weighed 1.76 tons? How many pounds of coal would it take to haul 1 passenger-train car a distance of 340.2 miles (Chicago to Cleveland)?

77. On Dec. 31, 1938, there was $6,851,199,897 in circulation in the United States. If the total population of the United States was 130,698,200 on that date, what would be the amount of money in circulation for each man, woman, and child?

78. One cubic foot of water weighs 62.5 pounds; find the volume of 1 pound of water; of 23 pounds.

79. One cubic foot of ice weighs 57.5 pounds; find the volume of 1 pound of ice; of 49.3 pounds.

80. How many times as heavy as ice is water? How many times as heavy as water is ice?

81. There are 1,000,000 persons employed in the petroleum industry and their annual earnings are $1,500,000,000. What are the average annual earnings for each person? If this average person pays 0.3 of his earnings for rent, 0.28 of it for food, 0.12 of it for travel, and 0.08 of it for clothing; how much does he pay for each item?

82. The following around-the-globe records have been set: In 1522, 3 years, 1 month; in 1580, 2 years 9 months; in 1588, 2 years 1 month 21 days; in 1889, 72 days 6 hours 11 minutes; in 1890, 67 days; in 1901, 60 days 13 hours; in 1913, 35 days 21 hours 36 minutes; in 1926, 28 days 14 hours 21 minutes; in 1931, 8 days 15 hours 51 minutes; in 1933, 7 days 18 hours

49 minutes; in 1938, 3 days 19 hours 8 minutes. Reduce all the records to decimals of a year, and find how many times longer it took each of the others as compared with the 1938 record. Use 360 days for a year and 30 days for a month.

83. Using United States standard, the gage and thickness for sheet steel are as follows: No. 00, 0.34375 inch; No. 2, 0.265625 inch; No. 4, 0.234375 inch; No. 7, 0.1875 inch; No. 13, 0.09375 inch; No. 28, 0.015625 inch. Find the approximate thickness of each in a common fraction of an inch having 8, 16, 32, or 64 for a denominator.

84. The commission merchants of Chicago receive 150 carloads of Christmas trees each December. If each car holds 1800 trees and the average price per tree is \$0.75 delivered to the final buyer, what gross amount will the merchants make if they pay \$160,000 for the trees?

85. A merchant sold 56 topcoats for \$24.25 apiece, 38 more at \$27.50 apiece, 20 more at \$33.75 apiece, and 15 more at \$44.25 apiece. What was the total selling price for all the coats?

86. Number 8 (B. & S.) gage sheet steel is 0.1285 inch thick and weighs 5.22 pounds per square foot. (*a*) Find the thickness of a pile of 48 such sheets. (*b*) Find the nearest whole number of sheets to make a pile 1 foot thick. (*c*) Find the weight of this number of sheets if each sheet has 6.25 square feet.

87. Number 25 (B. & S.) gage sheet copper is 0.0179 inch thick and weighs 0.811 pound per square foot. Answer the same questions as in Exercise 86.

88. An iron chain made of $1\frac{3}{4}$-inch round iron has a breaking strain of 88,301 pounds. If the chain weighs 17.5 pounds per foot, how long would the chain have to be to break of its own weight if suspended from one end?

89. Answer the same question as in Exercise 88 for a chain made of $\frac{5}{16}$-inch round iron, the chain weighing 0.994 pound per foot and breaking under a strain of 4794 pounds.

90. How much must be paid for 1600 feet of steel bar weighing 1.87 pounds per foot and costing \$4.65 per hundred pounds?

91. If a steel tape expands 0.00016 inch for every inch when heated, how much will a tape 100 feet long expand?

92. A round piece of work being turned in a lathe is 2.2448 inches in diameter. What is the diameter after a cut $\frac{1}{32}$ inch deep is made on the work?

93. The area of the cross section of a steel wire is 0.976 square inch. If a total load of 79,056 pounds was applied to this wire, what would be the load per square inch of cross section? If the wire was 8 inches long and was stretched 0.70 inch, what was the elongation (stretch) per inch in length?

94. The cost of making an electric motor was reduced from \$6.25 to \$4.65. How much would be saved in cost to manufacture 12 dozen of these motors?

95. A man has three rooms to carpet. One room is 10 feet by 12 feet, another is 12 feet by 16 feet, and the third is 6 feet by 8 feet. If the carpet is \$3.60 per square yard, what will be the cost to carpet the three rooms? (9 square feet equal 1 square yard.)

96. The formula for determining the number of threads per inch on machine screws is

$$N = \frac{6.5}{A + 0.02}$$

where

N = the number of threads per inch

A = the diameter of the screw

Compute the number of threads per inch for machine screws of the following diameters: 0.242; 0.398; 0.450; 0.563; 0.750. In each case give the answer to the nearest whole number.

97. A drill makes 280 R. P. M. and drills a hole $2\frac{1}{8}$ inches deep in $\frac{4}{5}$ minute. What is the feed? Feed is the distance the drill advances for each revolution.

98. A drill $1\frac{1}{4}$ inches in diameter makes 120 R. P. M. and has a feed of 0.012 inch. How long will it take to drill 16

holes in a cylinder head $1\frac{1}{2}$ inches thick, if one minute is allowed for setting for each hole?

99. The area of a pop safety valve to be placed on a steam boiler is determined by the formula

$$A = \frac{22.5 \times G}{P + 8.62}$$

where

A = area in square inches.

G = area of grate surface in square feet.

P = gage pressure in pounds.

Find the area of a safety valve on a boiler with 42 square feet of grate surface and carrying 120 pounds of gage pressure.

100. The dimensions of a key for a shaft as shown in Fig. 10 are $a = 1.3$ times b which is 0.65 inch, and $d = \frac{1}{3}$ of c. Find a, c, and d.

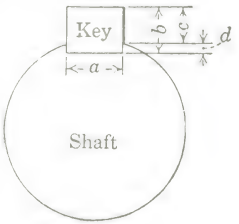

FIG. 10.

101. The relation between the voltage, current, and resistance in an electric circuit is

$$I = \frac{E}{R}$$

where

E = voltage measured in volts.

I = current measured in amperes.

R = resistance measured in ohms.

How many amperes of current will flow through the windings of an electromagnet of 132 ohms resistance when placed across a 110-volt circuit?

102. Four resistances connected in series across a 124-volt circuit have resistances of 2.2, 3.7, 6.4, and 5.3 ohms, respectively. Find the number of amperes of current flowing.

103. A Daniell cell has a voltage of 1.09 volts. How many amperes of current will this send through a circuit of 164 ohms resistance?

104. A lamp having a resistance of 75.2 ohms is in a 110-volt circuit. How many amperes of current does it use?

105. The power that is developed in an electric machine is expressed in watts or kilowatts. The power is determined by the formula $P = EI$, where P is the number of watts, E is the number of volts, and I is the number of amperes.

A dynamo furnishes power for the following at 110 volts: 15 electric heaters using 5 amperes each, and 250 incandescent lamps of 0.72 ampere each. Compute the load on the dynamo in kilowatts.

106. The work W done by an electric circuit is equal to the power P times the time T and is expressed by the formula:

$$W = PT$$

where

W = watt-hours or kilowatt-hours.

P = watts or kilowatts.

T = hours.

What is the cost to run a 110-volt, 600-watt electric heater for 8 hours at 3 cents per kilowatt-hour?

107. What is the cost of lighting a house for 30 days at 5.5 cents per kilowatt-hour if three 40-watt and five 60-watt lamps are used on an average of 3 hours a day?

108. A family uses a 550-watt toaster 15 minutes each day, five 40-watt and six 60-watt lamps an average of 3 hours a day, and a 660-watt electric iron 6 hours a week. What is the total cost for 4 weeks if the rate is 8 cents per kilowatt-hour for the first 24 kilowatt-hours, 6 cents for the second 24 kilowatt-hours, and 3 cents for the remainder?

109. The heat developed in an electric circuit is given by the formula

$$H = 0.24Pt$$

where

H = heat measured in calories.

P = power measured in watts.

t = time measured in seconds.

A calorie is the amount of heat required to raise the temperature of 1 gram of water 1 degree centigrade.

How much heat is generated in 20 minutes in an electric iron using 660 watts?

110. A certain electric corn popper uses 625 watts. If it takes 11 minutes to pop a batch of corn, how much heat is generated? How much does it cost to pop a batch of corn at 3 cents per kilowatt-hour?

111. The formula for finding the combined or equivalent resistance of several resistances connected in parallel is

$$r = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \cdots},$$

where r is the combined resistance and r_1, r_2, r_3, . . . are the resistances connected in parallel, all measured in ohms.

Three resistances of 6, 8, and 10 ohms, respectively, are connected in parallel; find the combined resistance.

112. Four lamps having resistances of 75, 75, 80, and 80 ohms, respectively, are connected in parallel; find the combined resistance.

113. Three lamps each having a resistance of 75 ohms are connected in series, and three other lamps each having a resistance of 80 ohms are connected in series. These two series are connected in parallel; find the combined resistance. When resistances are connected in series, the total resistance is the sum of the several resistances.

42. Proportions of machine screw heads. A. S. M. E. standard.—In the following four cases, the four standard heads are given. The proportions are based on and include the diameter of the screw, diameter and thickness of the head, width and depth of the slot, radius for round and fillister heads, and included angle of flat-headed screw.

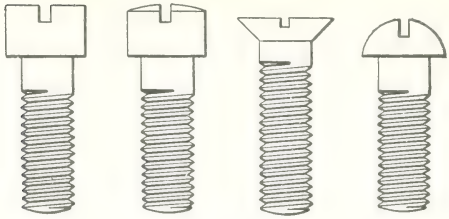

FIG. 11.

(1) *Oval fillister-head machine screws.* Fig. 12.

A = diameter of body.

$B = 1.64A - 0.009$ = diameter of head and radius of oval.

$C = 0.66A - 0.002$ = height of side.

$D = 0.173A + 0.015$ = width of slot.

$E = \frac{1}{2}F$ = depth of slot.

$F = 0.134B + C$ = height of head.

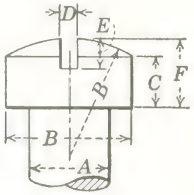

FIG. 12.

EXERCISES

Given the values of A, find those of B, C, D, E, and F:

	A	B	C	D	E	F
1.	0.216	0.3452	0.1406	0.052	0.093	0.1868
2.	0.398	0.6437	0.2607	0.084	0.173	0.3469
3.	0.450	0.729	0.295	0.093	0.196	0.3927

(2) *Flat fillister-head machine screws.* Fig. 13.

A = diameter of body.

$B = 1.64A - 0.009$ = diameter of head.

$C = 0.66A - 0.002 =$ height of head.
$D = 0.173A + 0.015 =$ width of slot.
$E = \frac{1}{2}C =$ depth of slot.

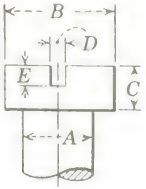

FIG. 13.

EXERCISES

Given the values of A, find those of B, C, D, and E:

	A	B	C	D	E
4.	0.112	0.1747	0.0719	0.034	0.036
5.	0.177	0.2813	0.1148	0.046	0.057
6.	0.320	0.5158	0.2092	0.070	0.105

(3) *Flat-head machine screws.* Fig. 14.
$A =$ diameter of body.
$B = 2A - 0.008 =$ diameter of head.
$C = \dfrac{A - 0.008}{1.739} =$ depth of head.
$D = 0.173A + 0.015 =$ width of slot.
$E = \frac{1}{3}C =$ depth of slot.

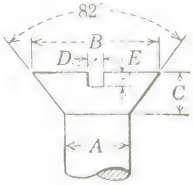

FIG. 14.

EXERCISES

Given the values of A, find those of B, C, D, and E:

	A	B	C	D	E
7.	0.086	0.164	0.045	0.030	0.015
8.	0.242	0.476	0.135	0.057	0.045
9.	0.372	0.736	0.209	0.079	0.070

(4) *Round-head machine screws.* Fig. 15.

A = diameter of body.

$B = 1.85A - 0.005$ = diameter of head.

$C = 0.7A$ = height of head.

$D = 0.173A + 0.015$ = width of slot.

$E = \frac{1}{2}C + 0.01$ = depth of slot

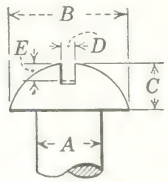

FIG. 15.

EXERCISES

Give the values of A, find those of B, C, D, and E:

	A	B	C	D	E
10.	0.073	0.130	0.051	0.028	0.035
11.	0.164	0.298	0.115	0.043	0.067
12.	0.398	0.731	0.279	0.084	0.149

THE MICROMETER CALIPER

43. The screw is used in very many mechanical devices. Some of these will be used in illustrative problems in later chapters. The use of the screw in measuring

small distances where great accuracy is required is illustrated in the ordinary *micrometer caliper* shown in Fig. 16.

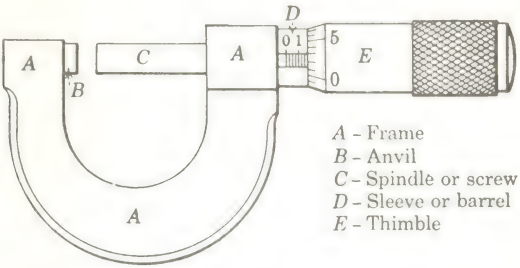

FIG. 16.

The object to be measured is placed between the **anvil** *B* and the **spindle** *C*. The spindle has a thread cut 40 to the inch on the part inside the **sleeve** *D*. The thimble *E* is outside the sleeve and turns the spindle. It also protects the thread from dust and wear. One complete turn of the thimble changes the opening of the micrometer by $\frac{1}{40} = 0.025$ inch, the same as the pitch of the thread.

The sleeve is graduated on a line along the length of the spindle into divisions of $\frac{1}{40}$ inch each, every fourth of which is marked 1, 2, 3, etc. The numbered marks then represent tenths of an inch.

The thimble has a beveled end that is divided into 25 equal divisions. A turn from one of these to the next evidently moves the spindle $\frac{1}{25}$ of $\frac{1}{40}$ inch or 0.001 inch.

When the end of the thimble is at 0 on the sleeve, the end of the spindle *C* should just touch *B*.

As shown in Fig. 16 the thimble has been turned away from the 0 point. To determine the complete reading, (1) note the reading on the sleeve as uncovered by the

thimble, which here shows 1 numbered division and 3 small divisions; (2) notice that the third division from 0 on the beveled edge of the thimble is on the center line of the spindle. From this we have:

1 numbered division	$=0.100$ inch
3 small divisions, 0.025 in. each	$=0.075$ inch
3 divisions on thimble	$=0.003$ inch
Complete reading	$=0.178$ inch

44. The vernier.—The **vernier** is a device, invented by *Pierre Vernier* (1580–1637), by which instruments can be read with a much greater degree of accuracy than is possible by mere mechanical division and subdivision. There are two kinds of verniers, known as the direct and reverse. Only the direct will be described here.

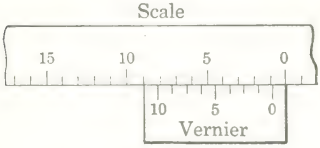

FIG. 17.

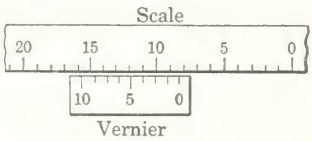

FIG. 18.

The principle is shown in its essentials in Fig. 17, which is a portion of a graduated scale, having below it a sliding scale, which is the vernier. The vernier is so divided that 10 divisions of its scale just equal 9 divisions of the graduated scale. If the 0 mark on the vernier coincides with a division, say the 0 division as in Fig. 17, of the graduated scale, then the division 1 on the vernier stands at 0.9 on the scale; 2 on the vernier at 1.8 on the scale; and so on for the other divisions.

If the vernier is moved along so that one of its divisions, as 4 in Fig. 18, coincides with a division of the scale, then the division on the vernier just to the right or left of the

coinciding division lacks 0.1 of a scale division of coinciding with a scale division. The next division of the vernier to the right or left lacks 0.2 of a scale division of coinciding with a scale division, and so on. In this case, the 0 point on the vernier is removed 0.4 of a division to the left of a scale division. The reading then in Fig. 18, that is, the distance from the 0 division on the scale to the 0 division on the vernier, is 7.4. If the scale division is tenths of an inch, the reading is 0.74 inch.

If the vernier is moved to the left so that 6 on the vernier coincides with a division on the scale, then 0 on the vernier is 0.6 of a scale division to the left of a scale division.

It is evident that any number of divisions on a scale could be equal to one greater number of divisions on a vernier, and the readings could be made in a similar way. For instance, in instruments for measuring angles, if the scale divisions are to $\frac{1}{2}^\circ$, then a vernier with 30 divisions equaling 29 divisions of the scale will give a reading of $\frac{1}{30}$ of $\frac{1}{2}^\circ$ or $1'$ of angle.

45. Micrometer with vernier.—A micrometer caliper that reads to thousandths of an inch may be made to read to ten-thousandths of an inch by putting a vernier on the sleeve, so that 10 divisions on the vernier correspond to 9 divisions on the thimble. There are eleven parallel lines on the sleeve occupying the same space as ten lines on the thimble. These lines are numbered 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. The difference between one of the ten spaces on the sleeve and one of the nine spaces on the thimble is $\frac{1}{10}$ of a space on the thimble or $\frac{1}{10000}$ inch in the micrometer reading.

In Fig. 19(*b*), the third line from 0 on the thimble coincides with the first line on the sleeve. The next

two lines do not coincide by $\frac{1}{10}$ of a space on the thimble, the next two, marked 5 and 2, are $\frac{2}{10}$ of a space apart, and so on. When the micrometer is opened the thimble is turned to the left and each space on the thimble represents $\frac{1}{1000}$ inch. Therefore, when the thimble is turned so that the lines 5 and 2 coincide, the micrometer is opened $\frac{2}{10}$ of $\frac{1}{1000}$ inch or $\frac{2}{10000}$ inch. If the thimble is turned farther, so that the line 10 coincides with the line 7 on the

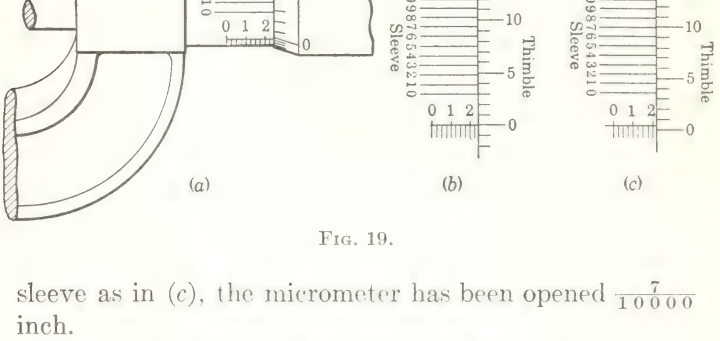

FIG. 19.

sleeve as in (*c*), the micrometer has been opened $\frac{7}{10000}$ inch.

To read a micrometer graduated to ten-thousandths, note the thousandths as usual, then observe the number of divisions on the vernier until a line is noted that coincides with a line on the thimble. If it is the second line, marked 1, add $\frac{1}{10000}$; if the third, marked 2, add $\frac{2}{10000}$, etc. Thus the reading for Fig. 19(*c*) is

0.2000 inch + 0.0250 inch + 0.0007 inch = 0.2257 inch

EXERCISES

1. Find the distance the spindle advances when the thimble makes 7 full turns and 17 divisions.

2. What is the measurement when the reading on the sleeve is 5 graduations and on the thimble 14?

3. The thimble of a micrometer which is fully closed is given 4 full turns and 15 thimble graduations. What is the opening of the micrometer?

4. Through how many full turns and graduations must the thimble of a micrometer be turned to open the micrometer 0.178 inch? 0.213 inch? 0.287 inch?

5. What is the setting on a micrometer caliper for each of the following measurements: 0.146 inch, 0.348 inch; 0.467 inch?

In the following settings of a micrometer caliper with a vernier, A is the reading of the numbered graduation on the sleeve, B is the reading of the unnumbered graduation on the sleeve, C is the thimble reading, and D the vernier reading. Determine the reading of each setting.

6. $A=1$, $B=3$, $C=5$, $D=6$.

7. $A=3$, $B=1$, $C=17$, $D=3$.

8. $A=4$, $B=0$, $C=15$, $D=7$.

9. $A=2$, $B=1$, $C=18$, $D=8$.

10. What is the setting on a micrometer caliper with a vernier for each of the following measurements: 0.2356 inch; 0.1875 inch; 0.6643 inch; 0.4792 inch?

CHAPTER IV

SHORT METHODS AND CHECKS

46. Contracted methods and approximate results.—As a rule the practical man does not need a large number of decimal places. The results of all measurements are at best only an approximation to the truth. The accuracy depends upon the instruments, the methods used, and upon the thing measured. All that is necessary is to be sure that the magnitude of the error is small compared with the quantity measured. It is clear that in a dimension of several feet, a fraction of an inch would probably not make much difference; but if the dimension was small, such an error could not be allowed.

The man in practical work uses instruments which are of such accuracy as to secure results suitable to his purpose. If he requires measurements accurate to 0.001 inch, it is not necessary for him in a computation to carry his work to 0.00001 inch. A good rule to go by is not to calculate to more than one more decimal place than measurements are made.

Thus, if a measurement of 3.265 inches is made and is to be multiplied by 3.1416, it is not necessary to multiply in the usual way, for then there would be seven decimal places, while the measurement was accurate to only three places. If the multiplication is performed by multiplying first by the left-hand figure of the multiplier, and then passing toward the right, we have the following forms:

Form in full	*Contracted form*
3.265	3.265
3.1416	3.1416
9795	9795
3265	3265
13060	1306
3265	32
19590	19
10.2573240	10.2572

It will be noticed that one more decimal place is retained than the desired number.

In a similar manner, division can be contracted. Suppose it is required to divide 0.04267 by 3.278, and secure an answer correct to four significant figures.

The division in the full and contracted forms is as follows:

0.042670000	3.278	0.042670	3.278
3278	0.013017	3278	0.013017
9890		9890	
9834		9834	
5600		56	
3278		32	
23220		24	
22946		22	
274		2	

Hence the result correct to four significant figures is 0.01302−.

It is easy to follow the method in obtaining the above, but it is hardly worth spending time upon unless one is to do much computing of this kind.

EXERCISES

Solve the following by contracted forms:

1. 3.14159×3.14159 correct to four decimal places.
2. $9{,}376{,}245 \div 3724$ correct to the unit's place.
3. $100 \div 3.14159$ correct to 0.01.
4. $87{,}659{,}734 \div 5467$ correct to five significant figures.
5. 45.8636×26.4356 correct to five significant figures.
6. 6.234×0.05473 correct to four significant figures.
7. 4.326×0.003457 correct to five significant figures.

SUGGESTIVE SHORT METHODS

47. Numerous short methods in multiplication and division can be given. If benefit is to be derived from them, they must be very carefully fixed in mind and used whenever occasion arises. One who does much computing will discover various timesaving short cuts. Of course, it is hardly worth while for one who does but little computing to give time to learning contracted methods, for such methods will not be remembered unless used frequently. The methods given here are in multiplication and division.

48. Multiplication.—(1) *To multiply a number by* 5, 50, 500, *etc.: Multiply by* 10, 100, 1000, *etc., and divide by* 2.

Why will this give the result?

Example: $7856 \times 50 = 785{,}600 \div 2 = 392{,}800$.

Multiply the following without using the pencil:

76×50	432×50	5.5×5
96×5	768×500	4.35×50
88×500	47×50	79.2×5000

(2) *To multiply by* 25, 250, *etc.: Multiply by* 100, 1000, *etc., and divide by* 4.

Why will this give the result?

Example: $32\times250=32{,}000\div4=8000$.

Multiply the following without using the pencil:

256×25	8956×25	728×250
74.92×250	492×2500	942.3×2500

(3) *To multiply a number* by 125: *multiply by* 1000 *and divide by* 8.

Why will this give the result?

Example: $848\times125=848{,}000\div8=106{,}000$.

Multiply the following:

920×125	4.76×125	72.88×125	55.5×125

(4) *To multiply a number by* $33\frac{1}{3}$, $16\frac{2}{3}$, $12\frac{1}{2}$, $8\frac{1}{3}$, *or* $6\frac{1}{4}$. *Multiply by* 100 *and divide by* 3, 6, 8, 12, *or* 16.

Example: $84\times8\frac{1}{3}=8400\div12=700$.

Multiply the following:

$48\times33\frac{1}{3}$	$42.6\times16\frac{2}{3}$	$32\frac{1}{2}\times16\frac{2}{3}$	$41\frac{3}{5}\times8\frac{1}{3}$
$96\times12\frac{1}{2}$	$3.97\times8\frac{1}{3}$	$33\frac{1}{4}\times33\frac{1}{3}$	$19\frac{1}{5}\times6\frac{1}{4}$
$72\times6\frac{1}{4}$	$4.76\times33\frac{1}{3}$	$98.76\times16\frac{2}{3}$	$27\frac{3}{5}\times12\frac{1}{2}$

This rule can be used easily in multiplying a number by $37\frac{1}{2}$, $62\frac{1}{2}$, $87\frac{1}{2}$, $83\frac{1}{3}$, and other fractional parts of 100 or 1000.

Multiply the following:

$24\times62\frac{1}{2}$	$35\times333\frac{1}{3}$	$42\frac{2}{5}\times62\frac{1}{2}$
$32\times87\frac{1}{2}$	$476\frac{1}{2}\times625$	$71\frac{3}{5}\times37\frac{1}{2}$
$36\times83\frac{1}{3}$	$672\times62\frac{1}{2}$	$47\frac{3}{5}\times333\frac{1}{3}$
$64\times37\frac{1}{2}$	$272\times87\frac{1}{2}$	$36\frac{2}{5}\times83\frac{1}{3}$

(5) *To multiply a number ending in* $\frac{1}{2}$, *as* $2\frac{1}{2}$, $4\frac{1}{2}$, $11\frac{1}{2}$, *by itself: Multiply the whole number by the whole number plus* 1 *and add* $\frac{1}{4}$ *to the product.*

Examples: $8\frac{1}{2}\times 8\frac{1}{2}=8\times 9+\frac{1}{4}=72\frac{1}{4}$ and $11\frac{1}{2}\times 11\frac{1}{2}=11\times 12+\frac{1}{4}=132\frac{1}{4}$.

The reason may be shown as follows:

$$3\frac{1}{2}\times 3\frac{1}{2}=3\times 3+3\times\frac{1}{2}+\frac{1}{2}\times 3+\frac{1}{2}\times\frac{1}{2}$$

But

$$3\times\frac{1}{2}+\frac{1}{2}\times 3=1\times 3 \text{ and } \frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$$

Hence,

$$3\frac{1}{2}\times 3\frac{1}{2}=3\times 4+\frac{1}{4}=12\frac{1}{4}$$

Multiply the following:

$5\frac{1}{2}\times 5\frac{1}{2}$	$12\frac{1}{2}\times 12\frac{1}{2}$	$20\frac{1}{2}\times 20\frac{1}{2}$	$18\frac{1}{2}\times 18\frac{1}{2}$
$7\frac{1}{2}\times 7\frac{1}{2}$	$14\frac{1}{2}\times 14\frac{1}{2}$	$25\frac{1}{2}\times 25\frac{1}{2}$	$150\frac{1}{2}\times 150\frac{1}{2}$
$9\frac{1}{2}\times 9\frac{1}{2}$	$16\frac{1}{2}\times 16\frac{1}{2}$	$40\frac{1}{2}\times 40\frac{1}{2}$	$59\frac{1}{2}\times 59\frac{1}{2}$

Putting in the decimal form, we have

$$8\frac{1}{2}\times 8\frac{1}{2}=8.5\times 8.5=72.25$$

Now removing the decimal point, we have

$$85\times 85=7225$$

Multiply the following:

7.5×7.5	135×135	505×505
12.5×12.5	95×95	615×615
11.5×11.5	155×155	925×925

49. Division.—By using the inverse operations to those given in the rules of the preceding article, numerous divisions can be performed.

(1) *To divide by* $33\frac{1}{3}$, *divide by* 100 *and multiply by* 3.

(2) *To divide by* 125, *divide by* 1000 *and multiply by* 8.

(3) *To divide by* $16\frac{2}{3}$, *divide by* 100 *and multiply by* 6.

Similar rules can readily be made for dividing by $12\frac{1}{2}$, $6\frac{1}{4}$, $8\frac{1}{3}$, 25, 250, $3\frac{1}{3}$, $333\frac{1}{3}$, $166\frac{2}{3}$, etc.

Examples: $84 \div 12\frac{1}{2} = (84 \div 100) \times 8 = 6.72.$

$9 \div 16\frac{2}{3} = (9 \div 100) \times 6 = 0.54.$

$32 \div 125 = (32 \div 1000) \times 8 = 0.256.$

$450 \div 6\frac{1}{4} = (450 \div 100) \times 16 = 72.$

$23 \div 250 = (23 \div 1000) \times 4 = 0.092.$

The multiplications in such problems can usually be performed without using the pencil.

Divide the following:

$800 \div 12\frac{1}{2}$	$492 \div 16\frac{2}{3}$	$720 \div 8\frac{1}{3}$
$37.6 \div 250$	$923 \div 33\frac{1}{3}$	$783 \div 12\frac{1}{2}$
$7.62 \div 12\frac{1}{2}$	$436 \div 3\frac{1}{3}$	$7.29 \div 125$
$927 \div 333\frac{1}{3}$	$43.9 \div 250$	$8927 \div 166\frac{2}{3}$

50. Checking.—No check can be made that is absolutely certain to detect an error, but there are many very useful devices for checking the accuracy of the work.

(1) *Addition.*—A simple way to check addition is to re-add, taking the figures in some other order. Adding first up and then down, is very satisfactory.

(2) *Subtraction.*—An error in a subtraction will generally be detected by adding the remainder to the subtrahend. If this gives the minuend, the work is correct.

Example.—Minuend 37249

Subtrahend 18496

Remainder 18753

37249 = *subtrahend* + *remainder.*

(3) *Multiplication.*—A good way to check multiplication is to interchange the multiplicand and multiplier and multiply again.

A very convenient and quick method is to proceed as follows:

(*a*) Add the digits in the multiplicand. If this sum has more than one digit, add these. Continue till a number of one digit is found.

(*b*) Add the digits of the multiplier as directed in (*a*).

(*c*) Multiply together the numbers obtained in (*a*) and (*b*), and add digits till a number of one digit is found.

(*d*) Add digits of product as directed in (*a*).

(*e*) Compare results of (*c*) and (*d*). If they are the same the work checks.

Example

34768 *Multiplicand*
492 *Multiplier*

69536
312912
139072

17105856 *Product*

Check

Summing digits as directed in (*a*):

(*a*) for the multiplicand gives 1,

(*b*) for the multiplier gives 6.

$$6 \times 1 = 6$$

The sum of the digits from the product gives 6. Since this is the same as that obtained before, the work is checked.

(4) *Division.*—Division can be checked by multiplying the divisor by the quotient and then adding the remainder. The result should be the dividend.

A quicker way to check is to add the digits as directed for checking multiplication: (*a*) the dividend; (*b*) the

divisor; (*c*) the quotient; (*d*) the remainder. Multiply the results in (*b*) and (*c*), add the result in (*d*), and then add the digits in this result which should give the same as the result of (*a*) if the work is correct.

Example

Dividend	4923567	476 *Divisor*
	476	10343 *Quotient*
	1635	
	1428	
	2076	
	1904	
	1727	
	1428	
	299 *Remainder*	

Check

Adding digits:
(*a*) for the dividend gives 9,
(*b*) for the divisor gives 8,
(*c*) for the quotient gives 2,
(*d*) for the remainder gives 2.

$$8 \times 2 + 2 = 18$$

The sum of the digits of $18 = 9$, which is the same as the sum in (*a*) and so checks the work.

The preceding rules apply as well to decimals as to whole numbers, but do not check the position of the decimal point.

EXERCISES

First multiply then divide the following and check as directed in the preceding article:

1. 4,356,785 by 9725.
2. 73,872 by 937.
3. 37,653 by 3647.
4. 30,987 by 4098.
5. 75.859 by 746.
6. 546.89 by 37.94.
7. 1294 by 5.48.
8. 46,378 by 9.0489.
9. 50.05678 by 0.193.
10. 29.29087 by 3.1416.

CHAPTER V

WEIGHTS AND MEASURES

51. English system.—The English system of weights and measures is the one in common use in the United States. The most used tables and equivalents of this system follow. The problems which are given later are inserted as material for review of work which it is supposed the student has done previously.

(1) *Measures of time:*

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
365 days	= 1 common year (yr.)
366 days	= 1 leap year

(2) *Measures of length:*

12 inches (in. or ″)	= 1 foot (ft. or ′)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards	= 1 rod (rd.)
320 rods	= 1 mile (mi.)
5280 feet	= 1 mile
1760 yards	= 1 mile

(3) *Measures of area:*

144 square inches (sq. in. or $in.^2$)	= 1 square foot (sq. ft. or $ft.^2$)
9 square feet	= 1 square yard (sq. yd. or $yd.^2$)

$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd. or rd.^{2})
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

(4) *Measures of volume:*

1728 cubic inches (cu. in. or in.^{3})	= 1 cubic foot (cu. ft. or ft.^{3})
27 cubic feet	= 1 cubic yard (cu. yd. or yd.^{3})
128 cubic feet	= 1 cord (cd.)

(5) *Liquid measures:*

2 pints (pt.)	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
$31\frac{1}{2}$ gallons	= 1 barrel (bbl.)
231 cubic inches	= 1 gallon

(6) *Dry measures:*

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)
2150.42 cubic inches	= 1 bushel

It should be carefully noted that dry and liquid measures are very different. For instance, 4 quarts in liquid measure contain 231 cu. in., while in dry measure they contain 268.8 cu. in. approximately.

(7) *Measures of weight (avoirdupois):*

7000 grains (gr.)	= 1 pound (lb.)
16 ounces (oz.)	= 1 pound
100 pounds	= 1 hundredweight (cwt.)
2000 pounds	= 1 ton (T.)
2240 pounds	= 1 long ton

In practice it is customary to consider 1 cu. ft. of water as 62.5 lb. or 1000 oz.

EXERCISES

1. Reduce 27 yd. 2 ft. 11 in. to inches.

2. Reduce 18 hr. 20 min. 35 sec. to seconds.

3. Reduce 4 T. 7 cwt. 35 lb. 9 oz. to ounces.

4. Reduce 8 bu. 3 pk. 7 qt. 1 pt. to pints.

5. Reduce 12 A. 35 sq. rd. 6 sq. yd. to square yards.

6. Reduce 9732 sq. in. to higher denominations.

7. Multiply 16 cu. yd. 20 cu. ft. 1235 cu. in. by 8.

8. How many iron rails each 30 ft. long will be required to lay a railroad track from Chicago to Gary, 26 miles?

9. Find the value of a field 240 rd. long and 84.5 rd. wide, at $160 per acre.

10. Reduce 19 pt. to the decimal of a gallon.

11. How many steps does a man take in walking 4 miles 86 rd. if he goes 2 ft. 4.5 in. each step?

12. Find the weight of 1 gal. of water.

13. If common bricks are $2\frac{1}{8}$ in. thick, find the number of brick in a stack 4 ft. 3 in. high.

14. In the Indianapolis 1939 automobile race, 500 miles were covered in 4 hr. 20 min. 47.4117 sec. Find the average rate in miles per hour correct to the nearest 0.001 mile.

15. A wire 6 ft. 8 in. long was suspended with a weight of 250 lb. attached to one end. If the wire was stretched 0.356 in., find the stretch for each foot of wire.

16. One cubic inch of water equals what part of a pint?

17. How long will it take a pump delivering 2.75 gal. per stroke and making 84 strokes per minute to pump 500 bbl. of oil?

18. How many sacks each containing 2 bu. 2 pk. can be filled from a bin of wheat containing 360 bu.?

19. What decimal part of a foot is $\frac{1}{32}$ in.? 0.125 in.? What decimal part of a yard is each?

20. Reduce the following to decimal parts of a foot: 1 in.; 2 in.; $3\frac{1}{2}$ in.; $7\frac{3}{8}$ in.

21. Reduce each in Exercise 20 to a decimal part of a yard.

22. Reduce the following to decimal parts of a pound avoirdupois: $\frac{3}{4}$ oz.; $1\frac{1}{2}$ oz.; 3 oz.; $7\frac{1}{2}$ oz.; 13 oz.; $4\frac{1}{2}$ oz.

23. Reduce 39.37 in. to a decimal fraction of a rod.

24. Reduce a pressure of 14.5 lb. per square inch to ounces per square foot.

25. A clock that gains 40 sec. while it runs 8 hr. is correct on Sunday noon. What is the correct time when the clock registers noon one week later?

26. A car averages 30 mi. per hour. What does it average in feet per second?

27. If sound travels at the rate of 1090 ft. per second in air, in what time would the report of a gun be heard when fired at a distance of 0.218 mile?

28. New York City is 38 ft. above sea level and Niagara Falls is 570 ft. above sea level. The distance between these two cities is 461 miles. What is the average rise in inches for every foot distance between the cities?

29. A tank holding 7 bbl. has two pipes opening from it; one empties out 2 qt. in 5 sec., and the other 17 gal. per minute. How long will it take to empty the tank if both pipes are open?

30. A family uses 2 qt. of milk one day and 1 qt. the next. If they get 2 qt. June first and pay 5 cents a pint for it, what will be the milk bill for June?

31. A carload of potatoes has a total weight of 55,600 lb. The car alone weighs 15,675 lb. How many bushels of potatoes in the carload if potatoes weigh 60 lb. per bushel?

32. If a car averages 88 ft. per second, how many common years would it take to drive to the sun, which is 93,000,000 miles away?

33. Suppose the distance traveled by the earth about the sun to be 596,440,000 miles per year. What is the average

hourly distance traveled, taking the year to be $365\frac{1}{4}$ days? Find the average distance per second.

34. Find the area in acres of a farm which is represented on paper as a rectangle $3\frac{3}{4}$ in. by $10\frac{1}{2}$ in. on a scale of $\frac{1}{16}$ in. to the rod.

35. The total cost of making a cement walk 300 ft. long, 5 ft. wide, and 6 in. thick, where the cement was hand mixed, was as follows: Foreman, 8 hr. at $1 per hour; laborers, 120 hr. at a cost of $78.20; cement, $86; gravel, $34.08. Find the total cost per square yard and per square foot.

36. A farmer drew a load of potatoes to market for which he received 76 cents per bushel. If the wagon and load weighed 3710 lb. and the empty wagon weighed 1150 lb., find what he received for the potatoes. Sixty pounds of potatoes make 1 bu.

37. How many pounds of charcoal does it take to make 3 tons of gunpowder if the powder is $\frac{1}{16}$ sulphur, $\frac{3}{4}$ saltpeter, and the rest charcoal?

38. How many barrels of flour of 196 lb. each does it take to run a bakery 1 week of 7 days if the output is 6000 loaves a day, and there are $9\frac{1}{2}$ oz. of flour in each loaf?

39. (1) Find the number of cubic feet in a barrel to the nearest 0.001. (2) Find the number of cubic feet in a bushel to the nearest 0.00001.

40. A new copper cent weighs 48 gr. How many pounds will $50 in these weigh?

41. One of the largest diamonds in the world weighs $3025\frac{3}{4}$ carats. How many pounds avoirdupois is this, correct to the nearest 0.0001? A carat is 3.168 g.

42. If railroad ties are placed 18 in. apart from center to center, how many miles will 54,320 ties reach?

43. How many rails each 30 ft. in length are used in laying two railroad tracks from New York to Chicago, a distance of 870 miles. Find the weight of these rails at 110 lb. per yard.

44. Suppose that the distance from the earth to the sun is 91,713,009 miles, and that the sun's light reaches the earth in 8 min. 18 sec. What is the velocity of light per second?

45. The pressure of the atmosphere is 14.7 lb. per square inch. Find the pressure in pounds per square foot.

46. How high is a column of water to give a pressure of 1 lb. per square inch?

47. If the ends of an iron beam bearing 5 tons at its middle rest upon stone piers, it is required to find the necessary bearing surface of each pier if the stone will support 200 lb. per square inch of surface.

48. One voussoir (or block) of an arch ring presses its neighbor with a force of 50 tons. If the joint has a surface of 5 sq. ft., find the pressure per square inch.

49. Work is done when resistance is overcome. It is measured by the product of the force times the distance over which the force acts. As a formula this is $w = f \times s$, where w is the work, f the force, and s the distance. If the force is in pounds and the distance in feet, then the work is in foot-pounds.

A steam crane lifts a block of granite weighing 2 tons 80 ft. Find the work done in foot-pounds.

50. How many foot-pounds of work are necessary to pump 100 bbl. of water to a height of 120 ft.? Use 4.211 cu. ft. in 1 bbl. and 62.5 lb. to 1 cu. ft.

51. How many foot-pounds of work are done in lifting an elevator weighing 3 tons to the top of a building 220 ft. high? If the elevator is raised through this height in 2 min., how many foot-pounds of work are done per second? If an engine of one horsepower can do 550 foot-pounds of work per second, an engine of what horsepower will be necessary to lift the elevator to the top in 2 min.?

52. A man weighing 165 lb. ascends a stairway to a height of 60 ft. in 20 sec. How many horsepower does he exert?

52. Water pressure.—When water is stored in a tank, it exerts pressure against the walls of the tank, whether the walls are horizontal, vertical, or oblique. The force is exerted perpendicular to the wall in all cases. The pressure on a given area is equal to the weight of a column of water of that area in cross section and of height equal

to the distance that the given area lies below the surface of the water. This distance is spoken of as the **head.** Thus, the pressure on a square inch of the wall at a depth of 10 ft. is equal to the weight of a column of water 1 sq. in. in cross section and 10 ft. high.

If 1 cu. ft. of water weighs 62.5 lb., a column 1 ft. high and 1 sq. in. in cross section weighs 0.434 lb.

The following rule may then be used for finding the pressure per square inch at any depth:

Multiply the head of the water in feet by 0.434. *The result is the pressure in pounds per square inch.*

EXERCISES

1. What is the pressure per square inch on the circular bottom of a tank if the head of water is 45 ft.? What is the total pressure on the bottom of the tank if the area of the bottom is 3.1416 sq. ft.?

2. A closed tank the area of the bottom of which is 6 sq. ft. has a pipe leading upward to a reservoir. If the bottom of the tank is 75 ft. below the surface of the water in the reservoir, find the pressure on the bottom of the tank.

3. In the preceding exercise, suppose the tank is 2 ft. by 3 ft. and 8 ft. high. What is the total pressure on one of the larger sides? Here the average pressure per square inch is the pressure at a point halfway between the bottom and the top of the tank. What is the total pressure on the top of the tank?

4. A tank 3 ft. long and 2 ft. deep is full of water. What is the total pressure on one side? Does the width of the tank make any difference?

5. If it requires 500 lb. to hold a valve 2 sq. in. in area closed at the bottom of a tank, what is the head of water?

THE METRIC SYSTEM

53. From a study of weights and measures in the United States, it is seen that a legal standard, the troy

pound, has been established for the use of the mint; but that beyond that, our weights and measures in ordinary use rest on custom only with indirect legislative recognition. It is seen that the metric weights and measures are made legal by direct legislative permission; that standards of both systems have been equally furnished by the government to the several states; that the customary system has been adopted by the Treasury Department for use in the customhouses; but that the same department has by formal order adopted the metric standards as "fundamental standards" from which measures of the customary system shall be derived.*

Commercial intercourse between nations makes it advisable, if not necessary, to have a uniform system of weights and measures. Such relations cause those countries not already using the metric system to make more and more use of that system. For instance, large orders for locomotives placed in the United States by foreign governments or by corporations in those countries, have made it a matter of good business to carry out the manufacturing in metric units of measure.

The World War, beginning in 1914, made it necessary for large manufacturing plants in this country to make considerable use of the metric system. It is stated on good authority that the first six months following the entrance of the United States into the great war advanced the use of the metric system in this country more than had the previous ten years.

Manufacturing companies have for several years quoted sizes of reamers, drills, and other tools in the metric system as well as in the ordinary system. At present this custom is becoming increasingly more general.

* See *History of the Standard Weights and Measures of the United States* by LOUIS A. FISCHER. This may be secured from Superintendents of Documents, Government Printing Office, Washington, D. C., for 15 cents.

The terms *kilowatts*, *kilocycles*, *milliamperes*, and others used commonly in electrical work are akin to the metric system. Everyone is familiar with the use of the metric system in measuring radio waves.

54. Measure of length. The meter.—The length of the meter was at first determined as one ten-millionth part of the distance from the equator to the north pole. It was afterward found that there had been a slight error in this determination. At present the meter is the length at 0° centigrade of a certain bar made of 90 per cent platinum and 10 per cent iridium; this is called the *international meter* and is kept at the International Bureau of Weights and Measures near Paris.

The two copies of the meter which the United States has are made of the same material. One of these is used as the working standard, and the other is kept for comparison. To ensure still greater accuracy, these are compared at regular intervals with the international meter.

55. Legal units.—As has been stated, the Treasury Department has determined that the meter shall be the "fundamental standard" of length. By the act of July, 1866, Congress fixed the relation

$$1 \text{ meter} = 39.37 \text{ inches}$$

This is the only legal relation between the two systems, and is used in the Office of Standards of Weights and Measures in this country in deriving the inch, foot, yard, etc., from the meter. Determined in this way the customary units are legal.

In the Philippine Islands, Puerto Rico, and Guam the metric system is in general use, and is the sole legalized system for these islands.

56. Measure of surface.—There is no fundamental standard of surfaces or areas as there is of the measures of length. But as the measures of areas are based upon the units of length, and as these are standards, the measures of areas may be so considered.

57. Measures of volume, cubic and capacity measures. In the United States the fundamental standards of volume are: (1) the cubes of the linear units based on the international meter; (2) the liter, which is the volume of the mass of one kilogram of pure water at its greatest density; (3) the gallon, which is 231 cu. in.; (4) the bushel, which is 2150.42 cu. in. The liter here used is almost exactly 1 cu. dm., and the inch is derived from the meter according to the relation 1 m. = 39.37 in.

58. Measures of mass.—The fundamental standard of mass (weight) in the United States is the international kilogram, a cylinder of 90 per cent platinum and 10 per cent iridium, preserved at the International Bureau of Weights and Measures. As in the case of the meter, one of the two copies of the kilogram possessed by the United States is used as a working standard, and the other is kept under seal and used only to compare with the working standard from time to time. To ensure still greater accuracy, these are compared at regular intervals with the international kilogram.

By act of Congress of July 28, 1866, the pound is derived from the kilogram. The relation established at that time was 1 kilogram = 2.2046 pounds avoirdupois. This relation has since been made more accurate and is 1 kilogram = 15,432.35639 grains, which would change the first relation to 1 kilogram = 2.20462234 pounds avoirdupois, or 1 pound avoirdupois = 453.5924277 grams.

This value is the one used by the National Bureau of Standards in Washington. It is thus seen that the avoirdupois pounds, ounces, etc., in common use are derived from the kilogram and so are fixed and definite derived units.

The established relation between the troy pound and the avoirdupois pound is 1 troy pound $= \frac{5760}{7000}$ avoirdupois pounds.

When made, the standard kilogram was supposed to be the exact mass of one cubic decimeter or 1 liter of pure water at the temperature of its greatest density. It has been found that this is not exactly true, but the difference is very slight, the kilogram being about 25 parts in 1,000,000 too heavy. This difference is so very small that it could hardly affect any ordinary problem.

59. Historical note.—*Simon Stevin*, the inventor of decimal fractions, was very enthusiastic not only over decimal fractions, but also over the decimal divisions of weights and measures. He said that he considered it the duty of governments to establish a decimal system of weights and measures. He little knew that two hundred years would elapse before the origin of the metric system, and that now, three and a half centuries later, the two great English-speaking nations would still be using the old system with all its adherent waste in time and labor.

60. Terms used.—In our customary system, that of feet and inches, pounds and ounces, etc., we have about 150 different terms to remember and 50 different numbers, ranging all the way from 2 to 1728, which bear little relation to one another. In the metric system we have only 14 different terms and but a single number, and that is the number 10.

In the metric system the different terms used are the following:

meter—the unit of length.
liter—the unit of volume.
are—the unit of area for land.
gram—the unit of weight.
myria—which denotes 10,000.
kilo—which denotes 1000.
hecto—which denotes 100.
deca—which denotes 10.
deci—which denotes 0.1.
centi—which denotes 0.01.
milli—which denotes 0.001.

Terms which are sometimes used are:

millier—which denotes 1,000,000.
quintal—which denotes 100,000.
stere—which is 1 cubic meter.

To these might be added **micron** and **microgram.**

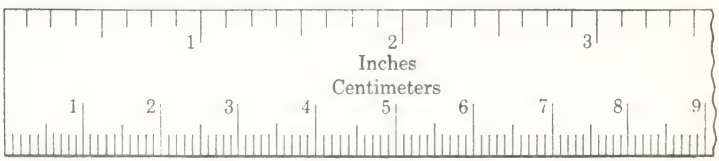

FIG. 20.

If the foregoing terms are carefully fixed in mind the tables are easily formed.

61. Tables.—In common usage but few terms in either our common system of weights and measures or in the metric system are used. In the following table, the terms most commonly used are printed in boldface type.

(1) *Measures of length:*

10 **millimeters** (mm.)	= 1 **centimeter** (cm.) = 0.01 m.
10 **centimeters**	= 1 decimeter (dm.) = 0.1 m.
10 decimeters	= 1 **meter** (m.)
10 **meters**	= 1 decameter (dkm.) = 10 m.
10 decameters	= 1 hectometer (hm.) = 100 m.
10 hectometers	= 1 **kilometer** (km.) = 1000 m.
10 **kilometers**	= 1 myriameter (mym.)
	= 10,000 m.

(2) *Measures of surface.*—In measuring areas squares determined by the measures of length are used, and we have square centimeters (sq. cm. or $cm.^2$), square meters (sq. m. or $m.^2$), etc. In land measure 100 sq. m. is an are, pronounced är.

(3) *Measures of volume.*—The more common measures of volume are the cubic centimeter (cc. or $cm.^3$), cubic decimeter, and cubic meter.

(4) *Measures of capacity.*—The liter (l.), which is very nearly one cubic decimeter, is the common unit of capacity. It is larger than a liquid quart and smaller than a dry quart. (See Fig. 21.) From this are derived deciliters, dekaliters, hectoliters, kiloliters, etc. The kiloliter is then 1 cu. m. and is commonly used.

(5) *Measures of weight.*—A cubic centimeter of water under certain conditions weighs 1 gram (g.). The unit of weight commonly used is the kilogram (kg.), which is the weight of a liter of water and is approximately 2.2 lb.; 1000 kg. are a *tonneau* or a metric ton.

To these may be added the following used in scientific work:

1 micron (μ)	= 0.000001 m.
1 microgram (γ)	= 0.000001 g.

NOTE.—A chart showing very clearly the relations of the different measures can be secured by addressing the Bureau of Standards, Washington, D. C.

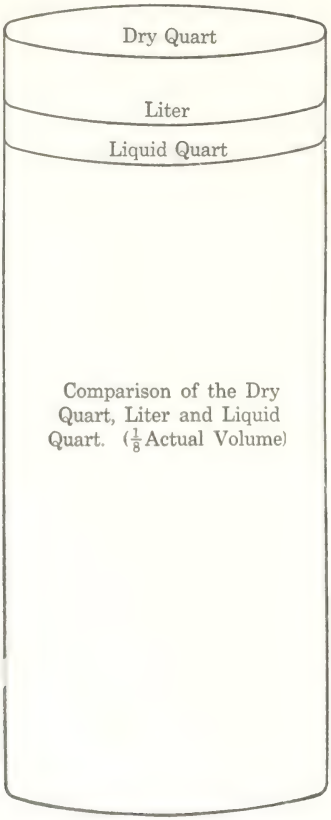

FIG. 21.

62. Equivalents.—The following equivalents enable one to change measurements in one system to those in

another. The first five should be remembered, the others are for reference.

1 m.	= 39.37 in. (established by law).
1 gram	= 15.432 grains.
1 lb. avoirdupois	= 7000 grains.
1 in.	= 2.54 cm. (approximately).
1 kg.	= 2.2 lb. (approximately).

Equivalents for Reference

Lengths

1 in.	= 2.54001 cm.
1 ft.	= 30.4801 cm.
1 km.	= 3280.83 ft. = 0.62137 mile.
1 mile	= 1.60935 km.

Areas

1 sq. in.	= 6.45163 $cm.^2$
1 sq. ft.	= 0.0929034 $m.^2$
1 sq. yd.	= 0.836131 $m.^2$
1 $cm.^2$	= 0.155 sq. in.
1 $m.^2$	= 10.76387 sq. ft. = 1.19599 sq. yd.
1 are	= 119.5985 sq. yd.
1 acre	= 40.4687 ares.

Volumes, capacities

1 cu. in.	= 16.38716 cc.
1 cu. ft.	= 28.317 l. or $dm.^3$
1 pt. (liquid)	= 473.179 cc. = 0.473179 l. or $dm.^3$
1 pt. (dry)	= 550.614 cc. = 0.550614 l. or $dm.^3$
1 qt. (liquid)	= 946.358 cc. = 0.946358 l. or $dm.^3$
1 qt. (dry)	= 1101.228 cc. = 1.101228 l. or $dm.^3$
1 $cm.^3$	= 0.0610234 cu. in.

1 liter	= 61.0234 cu. in.
1 liter	= 2.11336 pt. (liquid) = 1.81616 pt. (dry)
1 liter	= 1.05668 qt. (liquid) = 0.90808 qt. (dry).

Weights (mass)

1 grain	= 0.0647989 gram.
1 oz. (avoirdupois)	= 28.3495 grams.
1 lb. (avoirdupois)	= 453.5924277 grams = 0.45359+ kg.
1 ton (short)	= 907.185 kg.
1 g.	= 15.43235639 grains.
1 kg.	= 2.20462 lb. (avoirdupois).
1 metric ton	= 2204.62 lb. (avoirdupois).

63. Simplicity of the metric system.—The metric system was invented for simplicity.

Many look upon the system as difficult because they consider the difficulties of changing from the English to the metric system, or from the metric to the English, as difficulties of the metric system. All such difficulties would disappear if the metric system were in universal use, and considerable time would be saved in our schools that is now devoted to the teaching of our common system of weights and measures. Practically, where the two systems are in use, one or the other is used almost entirely, and one seldom needs to change from one system to the other. Exercises given in this text that require such changes are to help the student to visualize the relations between the systems. Figure 20 helps to visualize the relation between the inch and centimeter, and Fig. 21, the relation between the liter and the dry and liquid quarts.

The simpleness of the metric system lies in two facts: First, it is decimal, and therefore fits our decimal notation; second, its units for lengths, surfaces, solids, and weights are all dependent on one unit, the meter.

Ability to handle the metric system easily depends, in great part, on understanding thoroughly the terms used. It is of first importance then to learn well these terms and their meanings. For instance, the word *decimeter* should mean, at once, *one-tenth of a meter*.

Because of the decimal relations between the different terms used, the changing from one unit to another is a very simple matter. In reducing to higher denominations, we divide by 10, 100, 1000, etc., by removing the decimal point to the left.

Thus, to change 3768 cm. to meters, we divide by 100 by removing the decimal point two places to the left, and have

$$3768 \text{ cm.} = 37.68 \text{ m.}$$

In a similar manner 72,468 g. = 72.468 kg.

and

$$8643 \text{ l.} = 86.43 \text{ hl.}$$

It should be noticed that we never write 4 km. 7 hm. 3 dm. 5 m., but write it 4735 m. The former way of writing it would be similar to writing \$7.265 in the form 7 dollars 2 dimes 6 cents 5 mills.

In reducing to lower denominations, the multiplication is performed by removing the decimal point to the right.

Thus 25 m. = 250 dm. = 25,000 mm., and 16 kg. = 16,000 g.

64. Relations of the units.—It cannot be impressed upon the mind of the student too strongly that he

should understand clearly the relations between the units of different kinds of measure. He must know that a liter is a cubic decimeter, that a kilogram is the weight of a liter of pure water, that an are is a square dekameter, etc. He should notice that in the surface measures, when using square meters, dekameters, etc., the scale is 100; while in using cubic meters, dekameters, etc., for volumes, the scale is 1000.

Thus

$$2 \text{ m.}^2 = 200 \text{ dm.}^2 = 20{,}000 \text{ cm.}^2$$

and

$$3 \text{ m.}^3 = 3000 \text{ dm.}^3 = 3{,}000{,}000 \text{ cc.}$$

65. Changing from English to metric or from metric to English system.—Changing from one system to another is simply a matter of multiplication or division.

(1) Thus, to express 17 m. in inches,

$$1 \text{ m.} = 39.37 \text{ in.}$$
$$17 \text{ m.} = 39.37 \text{ in.} \times 17 = 669.29 \text{ in.}$$

(2) Also, to express 2468 lb. in kilograms,

$$2.2 \text{ lb. (approximately)} = 1 \text{ kg.}$$
$$2468 \text{ lb.} = 2468 \div 2.2 = 1121.8 \text{ kg.}$$

Or using the equivalent 1 lb. = 0.45359 kg.

$$2468 \text{ lb.} = 0.45359 \text{ kg.} \times 2468 = 1119.46 \text{ kg.}$$

The disagreement in the results is due to the fact that 2.2 lb. is a rough approximation.

The United States Bureau of Standards has compiled numerous tables of equivalents for use in the custom-houses. By the use of these tables, a conversion from

one system to another, is made by simply referring to the proper table and reading the result.

EXERCISES

1. Express the following, first, in meters and, second, in millimeters: 456 cm.; 1763 dm.; 27 km.

2. Express the following in square meters: 75 cm.^{2}; 125 mm.^{2}, 0.025 dm.^{2}; 0.0029 km.^{2}

3. Radio waves travel at approximately 186,000 miles or 300,000,000 m. per second. Do these agree?

4. A radio station that is broadcasting on a frequency of 720 kilocycles is using what wave length? Use the approximate velocity 300,000,000 m. a second.

5. Express the following in terms of cubic meters: 1756 l.; 467 kl.; 4937 dl.; 1067 dkl.; 735,432 dm.^{3}; 764 dkm.^{3}

6. Reduce 750 l. to liquid quarts; 326 l. to dry quarts; 75 m. to inches; 576 cm. to feet; 27 m.^{3} to bushels; 9276 mm.^{3} to gallons; 12 dm.^{3} to barrels.

Solution.—From Art. 62 we find 1 l. = 1.05668 qt. (liquid).

$$\therefore \quad 750 \text{ l.} = 1.05668 \text{ qt.} \times 750 = 792.51 \text{ qt.}$$

In the fifth part, 27 m.^{3} to bushels, the change is not so easy from the equivalents given.

$$27 \text{ m.}^3 = 27{,}000 \text{ dm.}^3 \text{ or l.}$$
$$1 \text{ l.} = 0.90808 \text{ qt. (dry).}$$
$$\therefore \quad 27{,}000 \text{ l.} = 0.90808 \text{ qt.} \times 27{,}000 = 24{,}518.16 \text{ qt.}$$

Divide this by 32 because 1 bu. = 32 qt.

$$\therefore \quad 27 \text{ m.}^3 = 766.19 \text{ bu.}$$

7. Reduce 456 in. to meters; 43.5 ft. to centimeters; 327 gal. to liters; 92.87 qt. (dry) to liters; 756 bu. to cubic meters.

8. No. 16 gage sheet steel is $\frac{1}{16}$ in. thick and weighs 40 oz. per square foot. Find the thickness in millimeters (four

decimal places) and the weight per square meter in kilograms (two decimal places).

Solution.—To find the weight in kilograms per square meter, first find the weight of a square meter in ounces, and then change to pounds and to kilograms.

1 m.^{2} = 10.76387 ft.^{2}

$\therefore$ 1 m.^{2} weighs 40 oz. $\times$ 10.76387 = 430.5548 oz. = 26.9097 lb.

1 lb. = 0.45359 kg.

$\therefore$ 26.9097 lb. = 0.45359 kg. $\times$ 26.9097 = 12.20597 kg.

9. No. 24 gage sheet steel is 0.635 mm. thick and weighs 4.882 kg. per square meter. Find thickness in decimals of an inch (three decimal places) and weight per square foot in ounces.

10. Find the difference between $3\frac{15}{16}$ in. and 10 cm.

11. What is the diameter in inches of the bore of a French "75" gun? This means 75 mm. in diameter.

12. In international athletic contests the distances are given in the metric system. In ice skating, the time for 1500 m. was 2 min. 20 sec. At this rate find the time for 1 mile.

13. In describing the making of reinforced concrete the necessary pressure is given as 25 kg. per square centimeter. How many pounds is this per square inch?

14. Find in kilograms the weight of air in a room 10.5 m. by 8.3 m. by 4 m., air being 0.001276 times as heavy as water.

15. Find the weight in kilograms of the mercury in a tube of 1 cm.^{2} cross section and 760 mm. long, mercury being 13.596 times as heavy as water.

16. If a map is made on a scale of 1 to 60,000, how many kilometers do 79 mm. on the map represent?

17. If a person in breathing uses 0.25 m.^{3} of air a minute, how long will it take six persons to use the air in a room 6 m. long, 3.5 m. high, and 5.3 m. wide?

18. A block of stone weighs 7643 kg. A cubic decimeter of the stone weighs 2.7 kg. Find the volume of the block in cubic meters.

19. Find the weight in kilograms of the water that a tank 2 m. by 3 m. by 6 m. will hold.

20. Find the weight in pounds of the water a tank 6 ft. by 10 ft. by 18 ft. will hold, using 62.5 lb. as the weight of 1 cu. ft. of water.

21. How many millimeters are there in each of the following: $\frac{1}{8}$ in.; $6\frac{1}{2}$ in.; $4\frac{13}{16}$ in.; $12\frac{7}{8}$ in.? Check your results by equivalents given in Table II, page 168.

22. Find the capacity in liters of a rectangular tank 2 m. by 9 dm. by 8 dm.

23. What is the length of a centigram of wire, 255 mm. of which weigh 0.172 g.?

24. A liter of mercury weighs 13.596 kg.; how many cubic millimeters of mercury weigh 1 g.?

25. A man's height is 174 cm. What is his height in feet and inches?

26. Express the following barometer readings in centimeters: 29.9 in.; 30.0 in.; 30.1 in.; 30.2 in.

27. Express the following in inches to the nearest 0.01: 71.119 cm.; 73.659 cm.; 74.929 cm.

28. Cast copper being 8.8 times as heavy as an equal volume of water, what is the weight of 5 $cm.^3$?

29. A velocity of 32.2 ft. per second is how many centimeters per second?

30. A rate of 1 mile in 2 min. 6 sec. is how many kilometers per minute? How many meters per second?

31. A rate of 30 miles per hour is the rate of 1 kilometer in how many minutes?

32. A pressure of 14.7 lb. per square inch is how many grams per square centimeter?

CHAPTER VI

PERCENTAGE AND APPLICATIONS

66. Per cents as fractions.—To one who thoroughly understands fractions, percentage offers no new difficulties.

The words per cent mean by the hundred. The symbol % means per cent. Thus, 10% means 10 per cent or $\frac{10}{100}$ or 0.10. In a similar way:

$5\% = 0.05 = \frac{1}{20}$	$50\% = 0.50 = \frac{1}{2}$
$12\frac{1}{2}\% = 0.12\frac{1}{2} = \frac{1}{8}$	$60\% = 0.60 = \frac{3}{5}$
$16\frac{2}{3}\% = 0.16\frac{2}{3} = \frac{1}{6}$	$62\frac{1}{2}\% = 0.62\frac{1}{2} = \frac{5}{8}$
$20\% = 0.20 = \frac{1}{5}$	$75\% = 0.75 = \frac{3}{4}$
$25\% = 0.25 = \frac{1}{4}$	$80\% = 0.80 = \frac{4}{5}$
$33\frac{1}{3}\% = 0.33\frac{1}{3} = \frac{1}{3}$	$83\frac{1}{3}\% = 0.83\frac{1}{3} = \frac{5}{6}$
$37\frac{1}{2}\% = 0.37\frac{1}{2} = \frac{3}{8}$	$87\frac{1}{2}\% = 0.87\frac{1}{2} = \frac{7}{8}$
$40\% = 0.40 = \frac{2}{5}$	$90\% = 0.90 = \frac{9}{10}$

To change a fraction, such as $\frac{2}{5}$, to an equivalent form in per cent, simply reduce it to a fraction having 100 for a denominator.

Thus,

$$\frac{2}{5} = \frac{40}{100} = 40\%$$

or

$$\frac{2}{5} = \frac{2}{5} \text{ of } 100\% = 40\%$$

Similarly

$$\frac{7}{8} = \frac{7}{8} \text{ of } 100\% = 87\frac{1}{2}\%$$

A per cent expressed as $\frac{2}{5}\%$, does not mean $\frac{2}{5}$ but $\frac{2}{5}$ of 1%, which is the same as $\frac{2}{5}$ of $\frac{1}{100} = \frac{2}{500} = \frac{1}{250} = 0.004$. In the same way $\frac{3}{8}\% = \frac{3}{8}$ of $\frac{1}{100} = \frac{3}{800} = 0.00375 = 0.375\%$.

It should be carefully noticed that the sign % does the duty of two decimal places.

Thus, $0.05 = 5\%$, $0.0005 = 0.05\%$, $1.07 = 107\%$, and $4.33\frac{1}{3} = 433\frac{1}{3}\%$.

67. Cases.—The problems of percentage usually occurring are of the following forms:

(1) What is $37\frac{1}{2}\%$ of 720?

(2) 45 is what per cent of 450?

(3) 85 is $62\frac{1}{2}\%$ of what number?

These three forms can be stated in general terms if the following words are defined:

The number of which the per cent is taken is the **base.**

The number of per cent taken is called the **rate.**

The part of the base determined by the rate is the **percentage.**

The sum of the base and percentage is the **amount.**

The base minus the percentage is the **difference.**

The three problems now become the cases:

CASE I. *Base and rate given to find percentage.*

CASE II. *Base and percentage given to find rate.*

CASE III. *Percentage and rate given to find base.*

These three cases of percentage correspond to the three cases in multiplication; when any two of the numbers, multiplicand, multiplier, and product, are given, to find the third.

In multiplication, { *multiplicand* corresponds to *base*; *multiplier* corresponds to rate; *product* corresponds to *percentage* } *in percentage.*

Case I corresponds to: *multiplicand* and *multiplier* given, to find the *product.*

Product = multiplicand × multiplier.

Case II corresponds to: *multiplicand* and *product* given, to find the *multiplier.*

Multiplier = product ÷ multiplicand.

Case III corresponds to: *multiplier* and *product* given, to find the *multiplicand.*

Multiplicand = product ÷ multiplier.

68. Rules and formulas.—In the language of percentage these become:

Case I. *Percentage = base × rate.*

This may be written as a formula if b stands for base, p for percentage, and r for rate. The formula is

$$p = b \times r.$$

Case II. *Rate = percentage ÷ base.* The formula is

$$r = p \div b.$$

Case III. *Base = percentage ÷ rate.* The formula is

$$b = p \div r.$$

69. Solutions.—*Problem* (1), Art. 67, is solved thus:

By fractions: $37\frac{1}{2}\%$ of $720 = \frac{3}{8}$ of $720 = 270$.

By formula: Using the formula $p = b \times r$ gives the same result, for then $p = 720 \times 0.37\frac{1}{2} = 270$.

Problem (2). *By fractions:* 45 is what per cent of 450 means 45 is how many hundredths of 450, that is, some number of hundredths of 450 is 45.

Then 45 is $\frac{45}{450} = \frac{1}{10} = \frac{10}{100} = 10\%$ of 450.

By formula: $r = p \div b$ gives $r = 45 \div 450 = 0.1 = 10\%$.

Problem (3). *By fractions:* 85 is $62\frac{1}{2}\%$ of what number is the same as 85 is $\frac{5}{8}$ of what number. It is now a simple problem in fractions and may be reasoned thus:

If 85 is $\frac{5}{8}$ of some number then 17 is $\frac{1}{8}$ of that number, and 136 is $\frac{8}{8}$ of that number.

Hence, the number = 136.

By formula: $b = p \div r$ gives $p = 85 \div 0.62\frac{1}{2} = 136$.

EXERCISES

Do the following exercises orally if possible:

1. What is $\frac{1}{3}$ of 24? $0.33\frac{1}{3}$ of 24? $33\frac{1}{3}\%$ of 24?
2. What is $\frac{2}{3}$ of 45? $0.66\frac{2}{3}$ of 45? $66\frac{2}{3}\%$ of 45?
3. What is $\frac{1}{4}$ of 32? 0.25 of 32? 25% of 32?
4. What is $\frac{3}{4}$ of 16? 0.75 of 16? 75% of 16?
5. What is $\frac{1}{5}$ of 40? 0.20 of 40? 20% of 40?
6. What is $\frac{4}{5}$ of 60? 0.80 of 60? 80% of 60?
7. What is $\frac{1}{8}$ of 64? $0.12\frac{1}{2}$ of 64? $12\frac{1}{2}\%$ of 64?
8. What is $\frac{5}{8}$ of 72? 0.625 of 72? 62.5% of 72?
9. What is $\frac{1}{16}$ of 48? $0.06\frac{1}{4}$ of 48? $6\frac{1}{4}\%$ of 48?
10. What is 25% of 20? of 48? of 88? of 140?
11. What is $33\frac{1}{3}\%$ of 45? of 90? of 120? of 360?
12. What is 8% of 8? of 20? of 88? of 800?
13. What is 10% of 8? of 20? of 88? of 800?
14. 4 is what per cent of 8? of 16? of 20?
15. 9 is what per cent of 9? of 18? of 72?
16. 40 is what per cent of 60? of 480? of 800?
17. What per cent is 4 of 74? $7\frac{1}{2}$ of 24?
18. What per cent is $\frac{2}{5}$ of $6\frac{1}{2}$? $37\frac{1}{2}$ of 150?
19. 10% of what number is 4? 8? 15? 90?
20. 33.5% of what number is 6? 16? 25? 96?
21. $62\frac{1}{2}\%$ of what number is 5? 20? 60? 500?
22. $66\frac{2}{3}\%$ of what number is 2? 20? 80? 6000?
23. 20% from what number leaves 48?

Suggestion.—20% from a number leaves 80% of the number.

24. 10% from what number leaves 9?

25. 20% from what number leaves 16?

26. $33\frac{1}{3}$% from what number leaves 20?

27. 40 is 20% less than what number?

28. 20 is 40% less than what number?

29. 40 is 20% more than what number?

30. 20 is 40% more than what number?

31. $37\frac{1}{2}$ is $87\frac{1}{2}$% less than what number?

32. If apples that cost 5 cents per pound were sold 3 lbs. for a quarter, what part of the cost is gained? What per cent?

33. Gas was bought for 12 cents per gallon and sold for 16 cents per gallon. What part of the cost is gained? What per cent?

34. A dealer allowed \$200 for a car and sold it for \$250; find the gain per cent.

35. Find the gain per cent in each case if an old car was bought for the following prices: \$50, \$40, \$25, \$10, \$5, \$1; and sold for \$50. Find the gain per cent if the car was given to the seller.

36. A pair of shoes cost \$8. They were worn 1 year and sold for \$1. What was the discount in per cent?

37. My milk bill was \$10.20 last month; this month it is \$9.80. Find per cent discount.

38. My gas bill last month was \$2.50; this month there was a 20% discount. What was my bill this month?

39. The 1937 Dodge sold for \$907; the 1938 Dodge sold for \$970. Find the per cent increase.

40. A quantity of wool was bought for \$360, and three-fourths of it was then sold for the cost of the whole. What per cent would have been gained if the entire amount had been sold at the same rate?

41. A man spent $16\frac{2}{3}$% of his salary for board and room. If he spent \$12.50 a week for board and room, what was his yearly salary? (52 weeks in a year.)

70. Applications. *Example* 1.—The population of a certain city in 1935 was 52,600, and in 1940 was 61,805. Find the gain in population in the 5 years. What was the gain for each 100 of the population during the 5 years? State this increase as a per cent of the population in 1935. What was the average per cent of increase per year?

Discussion: $61{,}805 - 52{,}600 = 9205 =$ gain in 5 years. $9205 \div 526 = 17.5 =$ gain per 100 of the population. Since asking for the per cent of gain is the same as asking for the number of increase for each 100 of the population, therefore, stated in per cent, this is 17.5% of the population.

$17.5\% \div 5 = 3.5\% =$ average per cent of increase per year based on the population in 1935.

Example 2.—In a certain machine two-fifths of the energy supplied to the machine is lost in friction and other resistances. What is the per cent of efficiency? If one-eighth of the loss of energy is in a certain part of the machine, what per cent of the total loss is in this part?

Discussion.—If in a machine it is known that $\frac{2}{5}$ of the energy expended is wasted in frictional and other resistances, we say that 40% is wasted, meaning that $\frac{40}{100}$ is useless for doing work. This does not state the actual numerical amount of energy wasted; all it tells is that for every 100 units of work expended on the machine, 40 units disappear. Such percentages enable comparisons of different machines to be made. If one machine has an efficiency of 60% and another of 70%, we know that the second is 10% more efficient than the first. If we know that $\frac{1}{8}$ or $12\frac{1}{2}\%$ of the 40% loss is in a certain part, we have a percentage of a percentage. The solution of the problem is

$\frac{2}{5}$ = part of the energy lost,
$\frac{3}{5}$ = 60% = efficiency of the machine,

$\frac{1}{8}$ of $\frac{2}{5} = \frac{1}{20} = 5\%$ = loss in the particular part of the machine.

71. Averages and per cent of error.—The data for practical calculations are in many cases either the result of measuring quantities, or of experimental observations, and in each case are liable to error. To obtain a result which can be relied upon, a number of measurements or observations are taken and the average or mean result calculated.

The average, or mean result, is obtained by adding all the measured results together and dividing the sum by the number of them. This average may be accepted as an approximation to the truth. The error of any particular observation is obtained by finding the difference between it and the average. This error can often be most conveniently expressed as a per cent and is spoken of as the per cent of error. We always take the correct value, or in this case the average value, as the base.

Example.—In measuring the diameter of a steel rod with a micrometer, the separate measurements are: 0.3562 in., 0.3569 in., 0.3567 in., 0.3570 in., and 0.3565 in. Find the average measurement and the per cent of error in the largest and the smallest measurements.

Solution and discussion: 0.3562 in. + 0.3569 in. + 0.3567 in. + 0.3570 in. + 0.3565 in. = 1.7833 in.

1.7833 in. ÷ 5 = 0.35666 in. = average.

0.3570 in. − 0.35666 in. = 0.00034 in. = error in largest measurement.

Using the formula $r = p \div b$ gives

r = 0.00034 in. ÷ 0.35666 in. = 0.00095+ = 0.095% = per cent of error in largest measurement.

0.35666 in. − 0.3562 in. = 0.00046 in. = error in smallest measurement.

$r = 0.00046$ in. ÷ 0.35666 in. = 0.0013 = 0.13% = per cent of error in smallest measurement.

It should be emphasized that the per cent of error in any measurement is always found by using the correct measurement as the base and the error as the percentage.

72. List prices and discounts.—The prices of machines and materials, printed in catalogues and price lists, are usually subject to discounts. Often the discount is so large that the list price gives no idea of the actual cost. In preparing an estimate, it is necessary to know what discounts are given from a price list.

Discounts are usually given thus: 60% and 10% off or simply 60 and 10 or perhaps "sixty and ten." This does not mean a discount of 70%, but that a discount of 60% is first made and then a discount of 10% on the remainder.

Thus, if the list price is \$3.50 with 60% and 10% off, we find 60% of \$3.50, which is \$2.10. Then deduct this from \$3.50 leaving \$1.40. Now get 10% of \$1.40, which is \$0.14, and deduct it from \$1.40, which leaves \$1.26 as the actual cost.

Similarly we may have discounts of 40%, 10%, and 4%, or 40, 10, and 4 off. These are deducted in turn as with the two discounts.

EXERCISES

1. 48% of 2000 = ?
2. What is $\frac{5}{8}$% of \$24.40?
3. What is 62.5% of \$24.40?
4. 30 is $2\frac{1}{2}$% of what number?

5. What per cent of \$70 is \$12.50?

6. 10% of 30 is 3% of what?

7. A man had \$256 in a closed bank. If 65% was paid, how much did the man lose?

8. There was an 18% reduction in traffic deaths in the United States for 1938, compared with 1937, which was a saving of 7200 lives. What were the traffic fatalities for the year 1938 in the United States?

9. In addition to the 40,000 motor-vehicle deaths in a single year, there were 1,150,000 nonfatal injuries. What per cent of the total killed and injured was fatal?

10. Accidents of all types claimed 94,100 people in the United States for the year 1938. If 40,000 of these deaths are due to motor vehicles, what per cent of deaths by accidents is this? Of the 130 million people in the United States, what per cent die by accident each year?

11. A gas range is priced \$103. A reduction of 10% was made for the old stove, and an additional reduction of 6% on the new price was then given for cash. What did the range cost the buyer?

12. A United States savings bond increased in value from \$18.75 to \$19 the first year and from \$24.50 to \$25 from the $9\frac{1}{2}$ to the 10th year. What was the per cent increase the first year? From the $9\frac{1}{2}$ to the 10th year?

13. If the finance cost for 1 year on an unpaid balance of \$480 is \$28.80, what per cent was charged?

14. A rough casting weighs $45\frac{1}{2}$ lb.; and, after being finished in a lathe, it weighs $43\frac{1}{4}$ lb. The loss in finishing is what per cent of the weight in the rough?

15. A steel I beam expands 0.01% of its length when exposed to the sun. Find the increase in the length of an I beam 25 ft. 8 in. long.

16. The number of hairs on the human head is approximately as follows: Redhead, 88,000; brunette, 102,000; blond, 104,000. What per cent increase has the blond over the redhead? Over the brunette?

17. The rated speed of a motor is 1760 R.P.M. If it is actually running at 1745 R. P. M., what per cent is this below the rated speed?

18. A manufacturer determines the selling price of his manufactured articles by adding 27%, approximately, to the cost. Find the selling price to the nearest higher 50 cents of a machine that cost \$56.25 to manufacture.

19. As cause of death per 100,000 note the following: heart disease, 245; cancer, 108; cerebral hemorrhage 85; pneumonia 82; nephritis, 81. What per cent of deaths is each of the above? What per cent of the total deaths is caused by all five of the above?

20. The United States Naval Academy graduated 578 men in June, 1939. Of this total number 52 were dropped from active service as physically unfit. What per cent was dropped?

21. A firm increases the wages of its employees $12\frac{1}{2}$%. Find the wages of a man who was getting \$3.40 a day. Of a boy who was getting \$1.60 a day. A man now receives \$6.30 a day; what did he receive before the increase?

22. An airline company carried 31,233 passengers in May, 1938, and 47,722 in May, 1939. What was the per cent increase?

23. A stock was bought for $\$13\frac{5}{8}$ and sold three years later for \$44.125. What per cent was gained?

24. The usual allowance made for shrinkage when casting iron pipes is $\frac{1}{8}$ in. per foot. What per cent is this?

25. Water in freezing expands 9% of its volume. How many gallons of water are necessary to make 240 cu. ft. of ice?

26. The profits of a business this last year were \$14,656, and were 28% more than for the previous year. What were the profits the previous year?

27. According to tests for career and self-appraisal of high-school students, 105 out of 1000 say they want to become physicians. Of this number, five actually do become physicians. What per cent want to become physicians? What per cent actually do?

28. An iron meteorite found in 1908 weighed 3275 lb. If 91.63% of its weight was iron, and 7.33% nickel, what was the weight of the iron? Of the nickel? Of other materials?

29. If a man had invested $100 in 1929 at the following peak values, what would his original capital be worth in June, 1939;

Seat on the New York Stock Exchange now 10.1% of 1929 peak.

Farm values now 72.4% of 1929 peak.

Bank account in London now 96.2% of 1929 peak.

Common stock of Homestake Gold Mining now 533.3% of 1929 peak.

U. S. Government Bond now 107.4% of 1929 peak.

Dow-Jones average of 30 industrial stocks now 35.9% of 1929 peak.

30. The per capita income for 1938 averaged $480, for 1937 it was $537. What was the per cent decrease?

31. The highest per capita income for 1938 was The District of Columbia which was $1065. Among the lowest was Alabama and North Dakota with a per capita income of $225 each for 1938. What per cent increase has the District of Columbia over Alabama?

32. The national income for 1938 was $62\frac{1}{2}$ billion dollars. This was a decrease of 10% over 1937. What was the United States national income for 1937? If 80 billion dollars income per year would balance the present United States budget, what per cent increase would this be over the 1938 national income?

33. If a 9 by 12 rug valued at $66.30 was offered for sale at $32.85 plus an additional 20% reduction on the offered price for quick sale, what was the final selling price of the rug? What total per cent reduction was the final selling price over the first price of $66.30?

34. A car is priced $824 which includes a federal tax of 3%. What was the price of the car? What was the tax?

35. The 3% sales tax on a car is $19.95. What is the price of the car including the tax?

36. A dealer sold two cars at $775 each. On one he lost 25% and on the other he gained 25%. Did he lose or gain on the entire transaction and how much?

37. The following measurements of the diameter of a small steel shaft were made with a micrometer: 0.5677, 0.5674, 0.5671, 0.5678, and 0.5673 in. Find the per cent of error in the greatest and in the least measurement.

38. When rock is crushed or broken into fragments of nearly uniform size, it increases in bulk and has voids, or interspaces, of from 30 to 55 per cent of the whole volume. Find the number of cubic yards when crushed occupied by 1 cu. yd. of solid rock, if voids are 30%; 35%; 40%; 45%; 55%.

Suggestion.—The volume of the rock, 1 cu. yd., is 30% less than, or 70% of, the bulk of crushed rock. Using formula

$$b = p \div r \text{ gives } b = 1 \div 0.70 = 1.43 -$$

39. The best time required for a letter mailed in Chicago and delivered in Washington, D. C., is as follows: straight first class, 22 hr. 45 min.; airmail, 14 hr. 30 min.; and special delivery airmail, 12 hr. 6 min. By using special delivery airmail, what per cent of time is saved over straight first class? Over airmail?

40. In estimating the amount to charge per day for the use of a steam roller, a contractor has the following data: first cost of steam roller, $3000; money worth 6%; days actually worked per year, 100; depreciation in value of the machine, $200 per year. Find the price to be charged per day for use of the roller.

41. The actual cost of removing a cubic yard of rock in excavating a certain canal is $1.10. What price should be put in the estimate, if 12% is to be allowed for superintending, and 10% on the cost, including superintending, is allowed for profit?

42. A house depreciates in value each year at the rate of 4% of its value at the beginning of each year, and its value at the end of two years is $6451.20. Find the original value.

43. A house valued at $4000 rents for $27.50 a month. The repairs on the house each year amount to $40, and the taxes are

$17.50. What interest does the property pay on the investment, no allowance being made for change in value of house?

44. In making a certain machine, 750 lb. of iron are used at an average cost of 12 cents per pound. There are used, in the work on the machine, 20 hr. of time at 60 cents per hour, 7 hr. at 90 cents, and 4 hr. at 30 cents. If 20% is allowed on cost as profit, what is the selling price of the machine? What will it be listed at if sold at 30 and 5 off?

45. A manufacturer sold a suit of clothes to a retailer at a profit of 20% on the cost of making. The retailer sold the suit to a customer for $72 and made a profit of $33\frac{1}{3}$% on what it cost him. How much did the suit cost the retailer and what was the cost of making?

46. A grain dealer in Chicago had his agent in Duluth buy 6500 bu. of wheat at $1.05, 4000 bu. at $1.08, and 7000 bu. at $1.01, paying a commission of $1\frac{3}{4}$% for buying. The wheat was shipped to Chicago by water and an insurance policy, at $1\frac{1}{4}$% premium, was taken out to cover cost of wheat and commission. Find the amount of the policy and the premium.

47. An article is listed at $225, and sells at 40 and 10 off. How will the 40% discount be changed to offset an increase of 15% in cost of production?

48. The composition of white metal is to be 4 parts by weight of copper, 9 antimony, and 97 tin. Express these as per cents, and find the weight of each material required to make 2376 lb. of alloy.

49. A ton of coal from the Rock Island field has 11.57% moisture, and 6.27% of the dry coal is ash. How many pounds of ash in a ton of the coal?

50. If 2.346 g. of an ore give 0.362 g. of copper, what per cent of copper does the ore contain?

51. 2.3656 kg. of ore give 0.7 g. of gold and 2.5 g. of silver, find the per cent of each.

52. A merchant buys rubber door mats at $48 per dozen less discounts at 40%, 15%, and 5%. At what price each should he sell them in order that he may make 35%?

53. If the author gets 10% of the selling price of a book, how many books selling at 75 cents each must be sold to pay the author $117.30?

54. In a compound of two substances A and B, their weights are in the ratio of 1.3498 to 1. What is the per cent of each in the compound? (That is, 1.3498 parts of A to 1 part of B.)

55. Two substances A and B form a compound and have a total weight of 3.267 g. If the compound has 24.725% of A and 75.275% of B, find the weight of each substance in the compound.

56. Find the cost of a steam boiler listed at $500 subject to discounts of 40%, 10%, and 7½%.

57. The recorded measurement of a city block is 528 ft. By chaining carefully the length is 527.75 ft. Find the per cent of error in the recorded length. How wide is a man's lot recorded as 30 ft.?

58. A sample of nickel steel contained 24.51% of nickel and 0.16% of carbon. How much of nickel and carbon, respectively, in 2240 lb. of nickel steel?

59. If a 3½% nickel-steel rail is used to maintain a curve in a streetcar track, it lasts three times as long as carbon steel. How much will be saved per ton when one nickel-steel rail is worn out, if nickel steel costs $56 per long ton and carbon steel $28? It costs $2 a ton for laying, and the old rails are worth $16 per ton; also 20 cents per pound is realized on the nickel.

60. In an experiment to show the loss of pressure for different kinds of valves in water pipes, a globe valve in a 3-in. pipe caused the pressure to fall from 80 lb. to 41 lb. per square inch; while a gate valve caused a loss of pressure of 4 lb. per square inch. Find (*a*) the per cent of loss for the globe valve; (*b*) for the gate valve; (*c*) what per cent the loss through the gate valve is of the loss through the globe valve.

61. In an analysis of the best quality of crucible cast steel, the following was found: carbon 1.2%, silicon 0.112%, phosphorus 0.018%, manganese 0.36%, sulphur 0.02%, iron

98.29% Cu. Find the number of pounds of each substance if the total weight is 176.5 lb.

62. The mean effective pressure on the piston of a steam engine, found from the indicator diagram, was 59.75 lb. per square inch. The boiler pressure was 87 lb. per square inch. What per cent of the boiler pressure was the mean effective pressure?

63. The grade of a railroad track is given in per cent. A grade of 1% is a rise of 1 ft. in 100 ft. If a railroad has a constant grade of $1\frac{1}{4}$%, what is the rise in $3\frac{1}{2}$ miles?

64. The total rise in a $1\frac{3}{4}$% grade is 43.6 ft. Find the length of the track having this grade.

65. A railroad rises 112.7 ft. in $3\frac{1}{2}$ miles. Find the average grade.

INTEREST

73. Interest is money that is paid for the use of money. It is usually reckoned at a certain rate per cent per year. The base on which the interest is reckoned is called the **principal.** In percentage, the time did not enter, but in reckoning interest the time has to be taken into account. The interest on a sum of money for 1 *year* at a certain rate is the principal multiplied by the rate; for 2 *years*, it is twice as much; and for any period of time, it is the interest for 1 year multiplied by the time in years.

If p stands for principal, I for interest, r for rate per cent, and t for time in years, the interest is found by the formula

$$I = p \times r \times t$$

The **amount** A is the principal plus the interest.

Many short methods for reckoning interest can be given, but here it is not the intention to enter into them.

Example 1.—Find the interest and amount of $350 for 5 years at 4%.

$$I = p \times r \times t = \$350 \times 0.04 \times 5 = \$70$$
$$A = p + I = \$350 + \$70 = \$420$$

Example 2.—Find the interest on $750 for 2 years 7 months at 2%.

Here the time is $3\frac{1}{2}$ years, for in getting the time in years we must use 12 months for a year, 30 days for a month, and 360 days for a year.

$$\therefore \quad I = \$750 \times 0.02 \times 3\tfrac{1}{2} = \$38.75$$

It is usually best to use cancellation.

Thus,
$$\frac{750 \times 2 \times 31}{100 \times 12} = 38.75$$

Example 3.—Find the interest on $375 for 2 years 5 months 15 days at 2%.

Here the time is $\frac{885}{360}$ years.

$$\therefore \quad I = \$375 \times 0.02 \times \tfrac{885}{360} = \$18.44$$

EXERCISES

Find the interest and amount of each of the following:

1. $700 for 3 years at 4%.
2. $14.30 for 2 years 9 months at 2%.
3. $245.60 for 2 years 7 months 21 days at 4%.
4. $436.75 for 1 year 2 months 15 days at 5%.
5. $325.25 for 2 years 9 months 12 days at $3\frac{1}{4}$%.
6. $87.50 for 3 years 3 months at $3\frac{1}{2}$%.
7. $480 for 6 years 3 months at 15%.
8. $18.20 for 9 years 9 months at $5\frac{3}{4}$%.

9. A note for \$225 at 3% runs for 9 months. What is the amount of the note when due?

10. A note for \$390 at $3\frac{1}{2}\%$ runs for 3 years 6 months. What is the amount due?

11. A bought a house from B on a contract for \$10,000 and paid \$1500 on delivery Apr. 1. The terms of the contract require the payment of \$135 on the first of each month, which is to pay the interest on the balance of the principal at 6% for the previous month and the remainder is applied as a payment on the principal. Make a table showing the interest paid, the payment on the principal, and the balance for the first of each month from May 1 to Sept. 1, inclusive.

NOTE: 6% per year $=\frac{1}{2}\%$ per month. Thus, the interest due on May 1 is

$$I = (\$10{,}000 - \$1500)(0.005)$$
$$I = \$42.50, \text{ etc.}$$

CHAPTER VII

RATIO AND PROPORTION

74. Ratio.—There are several ways of stating the relation of one quantity to another. If the size or magnitude of the quantities is thought of, a very convenient way of comparing them is to state the ratio of one to the other.

The **ratio** of one number to another is the quotient of the first number divided by the second.

Thus, the ratio of \$6 to \$2 is 3, and may be stated in the form $\frac{\$6}{\$2}$ or \$6 : \$2. In either case it is read *the ratio of* \$6 *to* \$2.

From the idea of a ratio it is evident that we can state a ratio between two magnitudes only when the magnitudes are alike; that is, a ratio cannot be stated between such quantities as dollars and bushels.

The two numbers used in a ratio are called the **terms** of the ratio. The first one is named the **antecedent** and is the dividend; the second is named the **consequent** and is the divisor.

The ratio 2 : 3 is the **inverse** of the ratio 3 : 2.

Since a ratio in the form 4 : 3 is an indicated division or a fraction, the principles applying in division or to a fraction likewise apply to a ratio.

The expressions, *in the same ratio as, in the same proportion, proportionally,* and *pro rata* all have substantially the same meaning.

When it is said that \$20 is divided between two men in the ratio of 2 to 3, it is meant that one gets \$2 as often as the other gets \$3. That is, of each \$5, one gets \$2 and the other \$3. Hence, one gets $\frac{2}{5}$ of \$20 or \$8, and the other gets $\frac{3}{5}$ of \$20 or \$12.

The idea which we name ratio is very common in practical matters. A drawing, such as a blueprint, is to some scale; that is, say $\frac{1}{8}$ inch on the drawing represents 1 foot on the actual object that is pictured. Maps are always to some scale definitely shown on the map by a scale of miles or by some other means.

EXERCISES

1. Find the value of the following ratios: 10:2; 8:5; 18:2.5; 25 hr.:3 hr.; 9 ft.:2 ft.; 5.5:4.5; 8.8:16.

2. A room is 18 ft. by 12 ft. What is the ratio of its length to its width?

3. Two gear wheels have 90 teeth and 20 teeth, respectively. What is the ratio of the numbers of teeth?

4. One city has a population of 8000 and a second a population of 20,000. What is the ratio of their populations? What part is the first of the second? What per cent? How many times as large as the first is the second? What difference is there in the ideas involved in the questions?

5. Write the inverse ratios of the following: 7:5; 8:2.5; \$12:\$20; 4.5 yd.:45 yd.

6. Divide 80 apples between two boys in the ratio of 5:3.

7. If a man can fly from Chicago to New York in 3 hr. 55 min. and go by train in 16 hr., what is the ratio of flying time to train time?

8. Fifty-one students entered a class and 33 of them finished the work. What per cent finished? What is the ratio of the number that finished to the whole number?

9. A man and his son earn \$60 per week. The ratio of their earnings is 5:1. What does each earn per week?

10. Four men owning adjoining lots with frontages of 25, 35, 50, and 75 ft., respectively, on the same street are assessed a total of $1500 for paving. Find the amount each should pay.

11. The efficiency of a machine is the ratio of the output to the input and is usually expressed as a per cent:

$$\text{Efficiency} = \frac{\text{output}}{\text{input}}$$

The input in a motor is 6000 watts and the output 5300 watts. What is the efficiency? When the efficiency is expressed in per cent, it is still a ratio with 100 for the denominator, or the consequent.

12. If the efficiency of a machine is 91% and the input 98 horsepower, what is the output?

13. If the efficiency of a motor is 98% and the output 730 watts, what is the input?

14. The output of a machine is 1040 horsepower and the loss 100 horsepower. What is the efficiency?

15. In a divided electric circuit, the current flowing divides in the same ratio as the ratio of the resistances of the two parts. The two resistances of a divided circuit are 65 and 75 ohms, respectively; and the current is 14 amperes; find the current flowing in each part.

PROPORTION

75. A proportion is a statement of equality between two ratios. Thus, $2:3 = 4:6$ and 4 men : 8 men = $6 : $12, are proportions.

The first and last terms of a proportion are called the **extremes.** The second and third terms are called the **means.** In the first proportion above, 2 and 6 are the extremes and 3 and 4 the means.

There are fashions in mathematics as there are in millinery, but the changes are not so frequent. Two

and three centuries ago the *Golden Rule*, as it was called, was widely used; later, and reaching well into the last century, this was called the *Rule of Three*. It was considered the best method for solving a great variety of problems and occupied a considerable part of arithmetic. In later years this rule was called *proportion*, and was considered important till some thirty years ago. Today proportion is given a minor place in our arithmetics, but is still considered convenient in solving certain problems especially in chemistry. It will repay one to become familiar with the methods of proportion as it gives another tool with which to work.

The more modern form for a proportion is the fractional form. Thus, $\frac{2}{3}=\frac{4}{6}$ has exactly the same meaning as $2:3=4:6$. Either form may be read "2 is to 3 as 4 is to 6."

76. Principles and methods.—By inspecting several proportions, the following principles will be evident:

(1) *The product of the means of any proportion is equal to the product of the extremes.*

(2) *The product of the two means divided by either extreme gives the other extreme.*

(3) *The product of the two extremes divided by either mean gives the other mean.*

Example 1.—Find the value of h from the proportion $25:100=7:h$.

Solution.—Applying principle (2), $h=\frac{100\times 7}{25}=28$.

Example 2.—If 15 tons of coal cost \$63, what will 27 tons cost at the same rate per ton?

Solution.—Since the same relation holds between the cost prices as between the amounts of coal, the ratio of 15 tons to 27 tons must equal the ratio of \$63 to the

cost of 27 tons. Let x stand for the number of dollars that 27 tons cost, and we can then state the proportion

$$15:27=63:x$$

$$\therefore \quad x=\frac{27\times 63}{15}=113.40$$

$$\therefore \quad 27 \text{ tons cost } \$113.40$$

Example 3.—If 25 men can do a piece of work in 30 days, in how many days can 35 men do the same work?

Solution.—It is evident that 35 men can do the work in less time than 25 men; hence the ratio of the number of days is equal to the inverse ratio of the number of men. Using x for the number of days required.

$$35:25=30:x$$

$$\therefore \quad x=\frac{25\times 30}{35}=21\tfrac{3}{7}$$

Therefore, 35 men can do the work in $21\frac{3}{7}$ days.

Example 4.—An inclined plane as shown in Fig. 22 rises 38 ft. in 100 ft. Find the height h that it will rise in 28 ft.

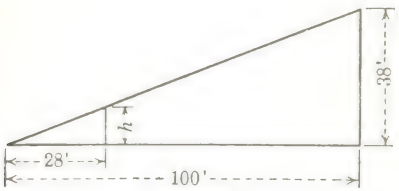

FIG. 22.

Solution.—Here the proportion is

$$100:28=38:h$$

$$\therefore \quad h=\frac{28\times 38}{100}=10.64$$

Therefore, the rise in 28 ft. is 10.64 ft.

The proportion could as well be stated

$$100:38=28:h$$

Definition.—If the rise of a roadbed is h ft. in 100 ft., the **grade** of the road is $\frac{h}{100}$, or the ratio of the rise to the *horizontal* distance. Thus, if a road rises 3 ft. in 100 ft., the grade is $\frac{3}{100}=3\%$.

Example 5.—What is the grade of a roadbed that rises 1.2 ft. in a horizontal distance of 40 ft?

Solution.—Let h stand for the number of feet rise in 100 ft. It is evident that the ratio $\frac{h}{100}$ = the ratio $\frac{1.2}{40}$. But $\frac{1.2}{40}=0.03$. Therefore, the grade is 0.03 or 3%.

Example 6.—If a bell metal is 25 parts copper to 12 parts tin, what is the weight of each in a bell weighing 1850 lb.?

Solution.—The ratio of the number of parts of each metal to the whole number of parts equals the ratio of the weight of each metal to the whole weight. Use c to stand for the number of pounds of copper and t for the tin. Then we have

$$25:37=c:1850$$

and

$$12:37=t:1850$$

$$\therefore \quad c=\frac{25\times 1850}{37}=1250$$

and

$$t=\frac{12\times 1850}{37}=600$$

Therefore, the weight of copper is 1250 lb. and weight of tin is 600 lb.

EXERCISES

Find the values of the letters in Exercises 1 to 6.

1. $70:45 = 14:x$.
2. $3\frac{1}{4}:7\frac{1}{8} = 15:x$.
3. $h:2.5 = 25:100$.
4. $75:x = 3\frac{1}{8}:\frac{1}{5}$.
5. $65:75 = x:85$.
6. $T:1.5 = 1.5:30$.

7. If a boat drifts down stream 40 miles in 12 hr., how far will it drift in 15 hr.?

8. If 5 men can do a piece of work in 40 days, how long will it take 25 men to do it?

9. On June 12, 1939, a pilot flew a glider plane across Lake Michigan a total distance of 92 miles in 52 min. He cut loose from the tow plane at 13,000 ft. and descended only 5000 ft. in crossing. At the same rate of descent, how much farther could he have glided? How many more minutes would he have been in the air?

10. A roadbed rises $3\frac{1}{3}$ ft. in a horizontal distance of 300 ft. What is the grade? In how many feet will it rise 1 ft.?

11. If 3 lb. of apples cost 25 cents, find the cost of 60 lb. at the same rate.

12. If 16 gal. of gas will drive a car 288 miles, at the same rate of using gas, how many gallons will it take to drive the same car from Chicago to Memphis, a distance of 564 miles?

13. The mixture for a casting has 4 parts of copper, 3 parts lead, and 2 parts tin. How many pounds of each in a casting weighing 96 lb.?

14. If the upkeep on 62 trucks for a year is \$3100, what would be the upkeep on 48 such trucks for 1 year at the same rate?

15. On a certain map $1\frac{1}{2}$ in. represent 50 miles. How many miles are between two cities that are 5 in. apart on the map?

16. An iron casting weighs 142 lb. and costs \$7.25. At the same rate, what is the cost of a casting weighing 255 lb.?

17. The number of revolutions of two gears in mesh is inversely proportional to the number of teeth in the gears. If

the number of teeth in the gear wheels is 15 and 48, respectively, and the R. P. M. of the smaller is 40, find the R. P. M. of the larger.

18. The volume of a quantity of gas is inversely proportional to the pressure upon it. If the volume of a quantity of gas is 740 cu. ft. under a pressure of 16 lb. per square inch, how many cubic feet will there be when under a pressure of 30 lb. per square inch?

19. A 10% solution of silver nitrate is formed by dissolving 10 grams of silver nitrate in 100 cc. of water. How many grams of silver nitrate should be used in 745 cc. of water to make a 10% solution?

20. A steel rail in a railroad track expands 0.00000636 in. for every inch of length for each degree increase in temperature. If a steel rail is 32 ft. long, what is the change in length due to a change from a winter temperature of 20 below zero to a summer temperature of 110 degrees above zero?

Work the following 4 exercises by proportion:

21. What per cent is 59.1 of 51.3?

Solution.—Let 100% stand for the base; then, using x for the number of per cent required, the proportion is

$$51.3 : 59.1 = 100 : x$$

$$\therefore \quad x = \frac{59.1 \times 100}{51.3} = 115.2$$

Therefore, 59.1 is 115.2% of 51.3.

22. 36 is what per cent of 80?

23. 248 is 21% of what number?

24. If 4% of my money is $2.88, how much money have I?

77. Measuring heights.—There are several methods for determining the height of a standing tree.

(1) One of the simplest is to measure the shadow of the tree and the shadow of a straight pole of known length set upright in the ground. Then if H stands

for the height of the tree, h for the height of the pole, S for the length of the shadow of the tree, s for that of the pole, we have the proportion

$$s:S=h:H$$

The two following methods with the figures are given in a bulletin of the Bureau of Forestry, U. S. Department of Agriculture.

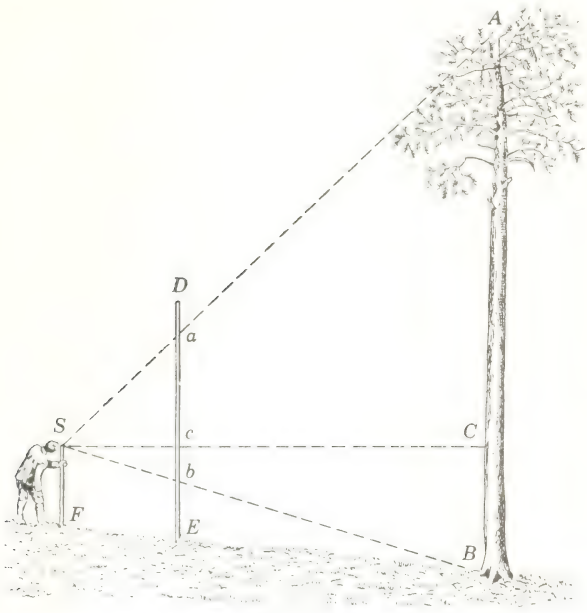

FIG. 23.

(2) A method used when the sun is not shining is to set two poles in a line with the tree as shown in Fig. 23. From a point S on one pole sight across the second pole to the base and to the top of the tree. Let an assistant

note the points a and b where the lines of vision cross the second pole and measure the distance between these points, ab, also measure the distance from the sighting point on the first pole to the base of the tree, SB, and to the lowest point on the second pole, Sb. Then the following proportion is true:

$$Sb : SB = ab : AB$$

(3) Another method sometimes used is as follows: The observer walks on level ground to a point A at a

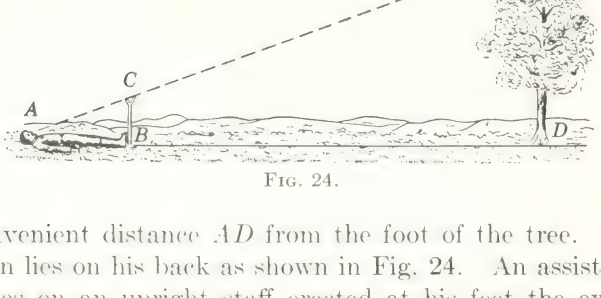

FIG. 24.

convenient distance AD from the foot of the tree. He then lies on his back as shown in Fig. 24. An assistant notes on an upright staff erected at his feet the exact point C where his line of vision to the top of the tree E crosses the staff. The height of the staff BC is measured, and his own height AB from his feet to his eyes, then the following proportion is true:

$$AB : BC = AD : DE$$

EXERCISES

1. Find the height of a tree that casts a shadow 115 ft. long when a pole 8 ft. high casts a shadow of 5 ft. Use method (1).

Solution: $5:8=115:H$.

$$\therefore \quad H=\frac{115\times 8}{5}=184$$

Therefore height of tree is 184 ft.

2. Find the height of a church steeple that casts a shadow 84 ft. long when a pole 11 ft. long casts a shadow of 7 ft. 9 in.

3. Find the height of a tree when $Sb=6$ ft., $SB=40$ ft., and $ab=9$ ft. Use method (2).

4. Find the height of the tree DE if $AB=5\frac{1}{2}$ ft., $BC=8$ ft., and $AD=90$ ft. Use method (3).

5. Find the height of a tree when $AB=5$ ft. 10 in., $BC=10$ ft., and $AD=112$ ft.

6. A tree 150 ft. tall stands on horizontal ground 600 ft. from a vertical cliff. A man whose eyes are $5\frac{1}{2}$ ft. from the ground, standing 200 ft. from the tree, sees the top of the cliff in line with the top of the tree. How high is the cliff?

78. Steam-boiler proportions.—Kent gives the following proportions of grate and heating surface required for a given horsepower in land boilers fired with good anthracite coal:

Heating surface per horsepower	11.5 sq. ft.
Grate surface per horsepower	$\frac{1}{3}$ sq. ft.
Water evaporated from and at 212° per square foot of heating surface per hour	3 lb.
Coal burned per horsepower hour	3 lb.
Coal burned per square foot of grate per hour	9 lb.

EXERCISES

1. Find the ratio of heating surface to grate surface and of grate surface to heating surface.

2. Find the number of pounds of water evaporated per square foot of grate surface per hour.

3. Find the number of pounds of water evaporated per pound of coal.

4. How many pounds of water does it take to run a 25-horsepower boiler 10 hr.?

5. Find the grate surface and heating surface of a 50-horsepower boiler.

79. The lever.—A stiff bar or rod supported at some pivotal point, about which it can move freely, is called a **lever.** The pivotal point is called the **fulcrum.** The lever enters in one form or another into many mechanical devices.

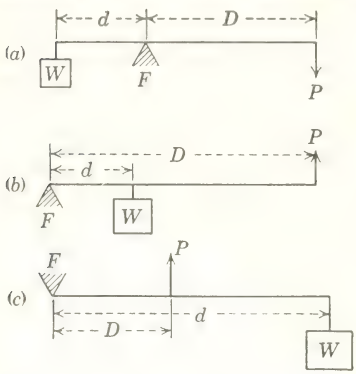

FIG. 25.

In Fig. 25, F is the fulcrum, W the weight lifted, P the force that does the lifting, D the distance from the fulcrum to the point of application of the force, and d the distance from the fulcrum to the point where the weight is attached. In all possible relations of the fulcrum, weight, and force the following proportion holds:

$$P:W=d:D$$

That is, the applied force is to the weight inversely as their distances from the fulcrum. This means that a small force will balance a larger weight only if the weight is nearer the fulcrum than the force.

EXERCISES

1. Given $P=150$ lb., $D=12\frac{1}{2}$ ft., $d=1\frac{1}{2}$ ft., find W.
2. Given $P=200$ lb., $D=9\frac{3}{4}$ ft., $W=775$ lb., find d.

3. Given $P = 160$ lb., $W = 900$ lb., $d = 1\frac{1}{4}$ ft., find D.

4. Given $W = 160$ lb., $D = 3\frac{1}{2}$ ft., $d = 8\frac{1}{3}$ ft., find P.

5. In a wire cutter the wire is placed $\frac{1}{2}$ in. from the fulcrum and the pressure of the hand is 7 in. from the fulcrum. Find the resistance of the wire if the hand exerts a force of 40 lb.

6. In pulling a nail from a board with a hammer, as shown in Fig. 26, find the resistance of the nail at the start if $P = 50$ lb., $D = 10$ in., and $d = 1\frac{1}{2}$ in.

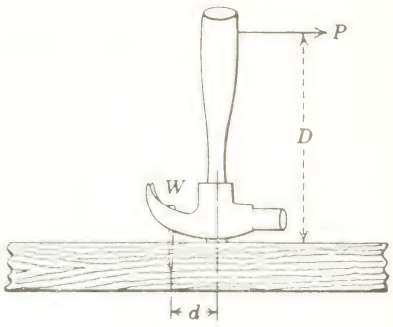

FIG. 26.

7. In the ordinary steelyard (Fig. 27) what must be the weight P to balance a weight W of $17\frac{1}{2}$ lb. if it is $1\frac{1}{4}$ in. from fulcrum to application of W and $8\frac{1}{2}$ in. from application of P to fulcrum?

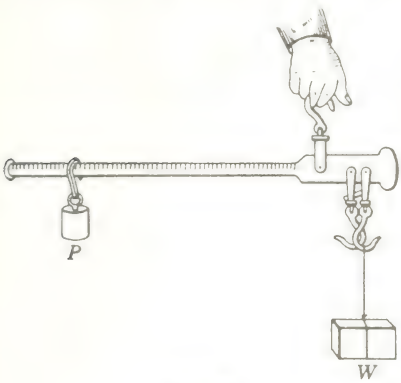

FIG. 27.

8. If the steelyard is turned over so that the distance from the fulcrum to W is $\frac{1}{2}$ in., what weight W will $1\frac{1}{2}$ lb. at P balance when P is $20\frac{3}{4}$ in. from the fulcrum?

9. In Fig. 28, what pull at B will balance a pull of 50 lb. at A?

80. Hydraulic machines.—A principle known as *Pascal's Law* states that pressure exerted on a liquid in a closed vessel is transmitted equally and undiminished in all directions.

In Fig. 29, if the area of a is 1 sq. in., then a pressure of 1 lb. at a gives a pressure of 1 lb. on each square inch of the surface of C. If the area of the top of C is 100 sq. in., then a pressure of 1 lb. at a will lift a weight of 100 lb. at A.

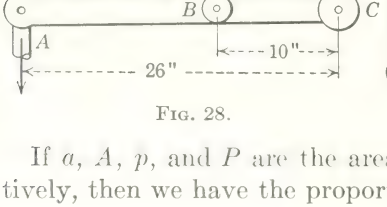

Fig. 28.

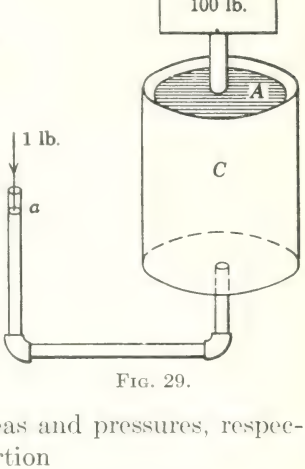

Fig. 29.

If a, A, p, and P are the areas and pressures, respectively, then we have the proportion

$$a : A = p : P$$

EXERCISES

1. A pressure of 5 lb. on the cork of a jug filled with water gives how many pounds pressure tending to force out the bottom of the jug? The area of the cork is $1\frac{1}{2}$ sq. in. and the area of the bottom is 245.6 sq. in.

2. A hydraulic lifter used to raise heavy weights, has the pressure applied to a piston having an area of $\frac{1}{2}$ sq. in. by a lever. The distance from the fulcrum to the point attached to the small piston is 4 in. and to the point where a force of 100 lb. is applied in 22 in. Find the weight that can be raised on a piston having an area of 75.6 sq. in.

3. A supply pipe for a 14-in. plunger hydraulic elevator piston is $1\frac{1}{4}$ sq. in. area, and the pressure in the supply pipe is pumped up to 150 lb. per square inch. What is the total pressure on the 14-in. plunger if it has an area of 153.94 sq. in.?

CHAPTER VIII

DENSITY AND SPECIFIC GRAVITY

81. Density.—Experience tells us that some bodies are heavier than others; that is, of two bodies of the same size, one weighs more than the other. Take a cubic foot of metal and one of wood; suppose the metal weighs 500 lb. and the wood 50 lb., then the metal is ten times as heavy as the wood, or the ratio of their densities is as 10 to 1. We also say that the density of the metal is 500 lb. per cubic foot.

Water has a density of about 62.5 lb. per cubic foot. In the metric system the density of water under standard conditions is one gram per cubic centimeter.

The **density** of a body is its mass per unit volume. For our purpose the mass is the same as the weight. Strictly speaking, the weight of a body near the earth is the force with which the earth attracts the mass of the body.

82. Specific gravity.—The term **specific gravity** is used for the ratio of the densities of two bodies. Thus, the specific gravity of the metal with reference to the wood is 10, which means that the metal is ten times as heavy as the wood. It should be carefully noticed that the specific gravity of a substance is an abstract number, that is, a number with no name attached.

83. Standards.—For convenience the standard to which other substances are referred, in stating specific gravities, is water for solids and liquids.

The specific gravity of a substance is obtained by finding the weight of a certain volume of it and dividing this weight by the weight of the same volume of the standard. Thus, to find the specific gravity of a stone, it is necessary to find its weight, and the weight of an equal volume of water. The weight of the stone divided by the weight of the water gives the specific gravity of the stone.

To find the specific gravity of a solid that is heavier than water, weigh it in air and then in water. Find the difference between its weight in air and its weight in water. This difference is the buoyant force of the water on the solid and is equal to the weight of the water displaced by the solid; that is, it is the weight of water having the same volume as the solid. Divide the weight of the solid in air by the difference, or the weight of an equal volume of water, and the ratio is the specific gravity of the solid.

The specific gravity of any other body could be found in the same manner. Some difficulty might be found in doing the weighing, but a little ingenuity will devise a plan. Various methods for doing the weighing are discussed in physics.

Water is taken as the standard because of its abundance. All substances can be referred to it, but gases are usually compared with air or hydrogen gas.

If w stands for the weight of the body whose specific gravity is to be found, s the weight of the same volume of the standard, and g for the specific gravity, the rule may be stated as a formula:

$$w \div s = g$$

84. Use.—Tables of the specific gravities of the various substances are given for use in making computa-

tions. In Table III, page 169, are given the specific gravities of a few of the more common substances.

If it is required to find the weight of a block of iron 2 ft. by 3 ft. by 1 ft., we could find the number of cubic feet in the block which is 6. This times the weight of a cubic foot of water gives the weight of an equal volume of water, or 62.5 lb.$\times$6=375 lb. The weight of the water multiplied by the specific gravity of iron gives the weight of the iron, or

$$375 \text{ lb.} \times 7.2 = 2700 \text{ lb.}$$

In terms of the letters already used, since $w \div s = g$,

$$w = s \times g$$

Example 1.—Find the specific gravity of a rock if 1 cu. ft. of it weighs 182 lb.

Solution.— Since water weighs 62.5 lb. per cubic foot the specific gravity of the rock is found thus,

$$182 \text{ lb.} \div 62.5 \text{ lb.} = 2.912$$

Therefore, specific gravity of the rock is 2.912.

Example 2.—How many cubic inches are there in 1 lb. of cork if its specific gravity is 0.24?

Solution.—Since 1728 cu. in. of water weigh 62.5 lb., 1728 cu. in. of cork weigh 0.24×62.5 lb.

Therefore, 1 cu. in. of cork weighs $\frac{0.24 \times 62.5}{1728}$ lb.

And

$$1 \div \frac{0.24 \times 62.5}{1728} = \frac{1728}{0.24 \times 62.5} = 115.2$$

Therefore, there are 115.2 cu. in. in 1 lb. of cork.

EXERCISES

1. Find the weights of the following: 12 cu. in. of rolled copper; 21 cu. ft. of cast iron; 231 cu. in. of nickel.

2. A body weighs 4 lb. in air and 2.5 lb. in water. Find its specific gravity.

3. What will the body in Exercise 2 weigh in alcohol of specific gravity 0.818?

4. A stone weighs 20 lb. in air and loses 20% of its weight when submerged in water. Find its specific gravity.

5. The specific gravity of a machine oil is 0.875. What would a 5-gallon can of this oil weigh? Use 7.5 gal. = 1 cu. ft.

6. What would 1 bbl. of 31.5 gal., or 4.2 cu. ft., of glycerine weigh if the specific gravity of glycerine is 1.26?

7. A 1000-cc. flask weighs 75 g., and when one half is filled with water and the other with glycerine, it weighs 1205 g. What is the specific gravity of the glycerine?

8. A body floats half-submerged in alcohol of specific gravity 0.818. What part of its volume would be submerged in water?

9. Given 1 cu. in. of lead and 1 of gold, find the ratio of their weights in air. What would be the ratio of their weights if both were submerged in water?

10. What is the specific gravity of a substance if 50 cu. in. of it weigh 8 lb. in air?

11. If the specific gravity of sea water is 1.025 and that of ice 0.92, what fraction of an iceberg floating in the sea is under water?

12. If a glass stopper weighs 150 g. in air, 90 g. in water, and 42 g. in sulphuric acid; what is the specific gravity of the acid?

13. A hollow steel ball weighs 1 kg. What must be its volume so that it will just float in water?

14. Find the number of liters in a vat 2 m. $\times$ 75 cm. $\times$ 50 cm. Also find the weight in kg. of the sulphuric acid (s. g. 1.84) required to fill it.

15. Find the value of 17 l. of sulphuric acid (s. g. 1.84) at 5 cents per kg.

16. Mercury weighs 13.596 times as much as water at its greatest density. What is the pressure per square centimeter of a column of mercury 76 cm. high?

17. A column of mercury how high would cause a pressure per square inch equal to 14.7 lb.?

18. A tank 1.85 m. long, 1.35 m. wide, and 85 cm. deep is filled with sea water (s. g. 1.025). What is the weight of the water?

19. Sandstone of specific gravity 2.5 is crushed. Find the weight of 1 cu. yd. of the crushed stone if the voids are 35%. (See Ex. 38, page 125.)

20. Granite of specific gravity 2.8 is crushed. Find the weight of 1 cu. yd. of the crushed rock if voids are 40%.

21. A casting of iron when immersed in water displaces 2 qt.; find the weight of the casting.

22. An irregular-shaped steel forging was found to displace 6.75 qt. of water; find the weight of the forging. (Use s. g. of steel = 7.85.)

23. A wooden pattern for a casting weighs $2\frac{3}{4}$ lb. An aluminum casting is to be made. Find the weight of the casting if the specific gravity of the wood is 0.52 and that of the aluminum is 2.6.

CHAPTER IX

POWERS AND ROOTS, SQUARE ROOT

85. Powers.—When we have several numbers multiplied together, as $3\times4\times6=72$, we call the numbers 3, 4, and 6 **factors** and 72 the **product.** Now if we make all the factors alike, as $3\times3\times3\times3=81$, we call the product by the special name **power.** We say that 81 is a power of 3 and 3 is the **base** of the power.

A **power** is a product obtained by using a base a certain number of times as a factor.

If the base is used twice as a factor, the power is called the *second power;* if three times as a factor, the *third power;* and so on for any number of times.

86. Exponent of a power.—Instead of $3\times3\times3\times3$, we may write 3^4. The small figure placed at the right and above the base shows how many times the base is to be used as a factor and is called an **exponent.**

The **exponent** of a **power** is a number placed to the right and above a base to show how many times the base is used as a factor.

It should be noted that the use of the exponent gives us a short concise way of writing a continued product where the factors are all alike.

87. Squares, cubes, involution.—The second power of a number is called the **square** of the number, as 3^2.

The third power of a number is called the **cube** of the number, as 5^3.

The higher powers have no special names; 3^4 is called the **fourth power** of 3; 5^7, the **seventh power** of 5; etc.

Involution is the process of finding the powers of numbers.

EXERCISES

1. Find the square of 6; of 21; of 38; of 606. Find the square of the square of 2; of 5; of 10.

2. Find the cube of 4; of 9. Find the square of the cube of 3.

3. Find the fourth power of 6. Is the fourth power of a number equal to the square of the square of the number? Find the value of 3^4; of $(3^2)^2$.

4. Find values of the following: 292^2; 25^3; 3^4; 1.2^5.

88. Roots.—If we take 9 and separate it into the two equal factors 3 and 3, that is, $9 = 3 \times 3$, then one of these factors 3 is called the **square root** of 9. The process is just the inverse of that by which the power is found. Similarly, $64 = 4 \times 4 \times 4$, and we say 4 is the **cube root** of 64.

The **square root** of a number is one of the two equal factors into which a number is divided.

The **cube root** is one of the three equal factors into which a number is divided; the **fourth root** is one of the four equal factors; and so on for the higher roots.

89. Radical sign and index of root.—To indicate a root, we use the sign $\sqrt{\ }$, which is called the **radical sign.** A small figure, called the **index** of the root, is placed in the opening of the radical sign to show what root is to be taken.

Thus, $\sqrt[3]{64}$ indicates the cube root of 64. The small 3 is the index of the root.

Since the square root is the most frequently written root, the index 2 is omitted.

Thus, the square root of 625 is written $\sqrt{625}$ and not $\sqrt[2]{625}$. Higher roots are indicated as $\sqrt[4]{243}$, $\sqrt[7]{128}$.

Evolution is the process of finding a root of a given number.

90. Square root.—The numbers 1, 4, 9, 16, 25, 36, 49, 64, 81, which are the squares of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, should be carefully remembered. It will be noted that these are the only whole numbers less than 100 of which we can find the square roots. Such numbers as these are called **perfect squares.** As we pass to numbers above 100, the perfect squares become still more scarce.

The square root of 49 is 7, but the square root of 56 cannot be expressed as a whole number, nor can it be expressed as a decimal exactly. We can, however, find it to any desired number of decimal places and therefore as accurately as we wish. It remains to devise a method by which this may be done.

The practical man who wishes to find the square root of a number does not care greatly why he goes through a certain process; but it is very important to him that he shall be able to find the root quickly and accurately. In what follows, then, the attempt is made to tell in as simple a manner as possible *how* to find the root.

91. Process for the square root of a perfect square. *Example* 1.—Find $\sqrt{522729}$.

Process

$52'27'29$ (723 *Root*

	49
142	327
	284
1443	4329
	4329

Explanation.—First, separate the number into **periods** of two figures each, beginning at the right, and placing a mark between them. The number of periods thus formed is equal to the number of figures in the root.

Find the largest perfect square which is equal to, or less than, the left-hand period 52. This perfect square is 49. Write it under 52; and put its square root, 7, to the right as the first figure of the root. Now subtract 49 from 52 and bring down the next period, 27, and unite with the remainder 3, thus obtaining 327.

Take twice 7, the first figure of the root, and write it to the left of 327. Find how many times this 14 is contained in 32, which is 2, for the second figure of the root. Place this figure 2 in the root, and also to the right of 14, making 142. Now multiply 142 by 2, and write the product 284 under 327. Subtract 284 from 327 and bring down and unite the next period, 29, with the remainder, 43, thus obtaining 4329.

In the above work 327 is called the **first remainder;** 14, the **trial divisor;** 142, the **true divisor;** and 4329, the **second remainder.**

Next multiply 72 by 2, and write it at the left of 4329 as the second trial divisor. Find how many times 144 is contained in 432, which is 3, for the third figure of the root. Place this figure 3 in the root and also at the right of 144, making 1443, the second true divisor. Multiply 1443 by 3 and write the product under 4329. This gives no remainder. Therefore, 723 is the exact square root of 522,729; that is, $723 \times 723 = 522{,}729$.

Example 2.—Find $\sqrt{6780816}$.

Process

6′78′08′16(2604 *Root*

	4
46	278
	276
5204	20816
	20816

Explanation.—First, separate the number into periods of two figures each as in Example 1. As before, we find the greatest square, 4, in the left-hand period, write it under 6 and put the square root, 2, of this square for the first figure of the root. Subtract the square, 4, from 6, and bring down the next period, 78, and unite it with the 2, making the first remainder 278.

Take twice 2 for a trial divisor. Find how many times it is contained in the first remainder, excepting the right-hand figure; that is, find how many times 4 is contained in 27. The number is 6, which write as the second figure of the root, and also at the right of the trial divisor. This makes 46 the true divisor. Multiply the true divisor by 6, and subtract the product, 276, from the first remainder. Bring down and unite the next period to the difference, making the second remainder, 208.

Multiply the root already found by 2, and get the second trial divisor, 52. Find how many times this is contained in 20, which gives 0 for the next figure of the root. Place this 0 in the root and also to the right of 52, making 520, the second true divisor. Now, since the 0 written in the root is the multiplier, nothing is gained by multiplying the true divisor by it, and subtracting from 208. This part of the process is omitted, and the next period, 16, is united with 208, making 20,816, the third remainder.

The third trial divisor is twice the root, 260, which gives 520. This is contained in 2081, 4 times. Place the 4 as the next figure of the root and also to the right of 520, making 5204, the third true divisor. Multiply this by 4 and subtract from the third remainder. Since the remainder is 0, 2604 is the exact square root of 6,780,816.

92. Square root of a number containing a decimal. *Example.*—Find $\sqrt{665.1241}$.

Process

```
      6'65.'12'41  (25.79 Root
       4
45     265
       225
507    4012
       3549
5149   46341
       46341
```

Explanation.—Here the division into periods is made by beginning at the decimal point and going in both directions. The rest of the work is the same as in Examples 1 and 2.

The student should note that the second trial divisor, 50, is contained 8 times in the first three figures of the second remainder, 4012. However, if 8 were used as the root, it would give a number larger than 4012 when the true divisor was multiplied by it. The relations noted here should help to make clear why we give to the trial divisor its name.

The decimal point in the root is so placed that there are as many whole number *figures* in the root as there are whole number *periods* in the number of which the root is extracted. The position of the decimal point can also be determined so that there will be as many decimal places in the root as there are decimal periods in the number of which the root is being extracted.

If the decimal part of the number consists of an odd number of figures, a 0 is annexed to make a full period at the right.

Thus, in pointing off 53.76542 into periods it is 53′.76′54′20.

93. Approximate roots.—Most numbers are not perfect squares, but the roots may be found to any desired number of decimal places. When extracting the root of a number not a perfect square, one must determine how many decimal places he wishes in the answer, and then annex zeros to the right of the number till there are as many decimal **periods** as there are to be decimal **places** in the root. The root is then extracted in the usual manner. We stop when the desired number of figures is found in the root.

Example.—Find $\sqrt{27}$ to three decimal places in the root.

Process

27.′00′00′00(5.196 *Root*

	25
101	200
	101
1029	9900
	9261
10386	63900
	62316
	1584

Explanation.—Since three decimal places are required in the root, annex three periods of zeros to the right of 27. These are the decimal periods. Extract the root as before. Place the decimal point in the root as in the

example in Art. 92. It will be noticed that there is a remainder. This is disregarded for it affects the next figures only, that is, the fourth and following figures in the decimal part of the root.

94. Root of a common fraction. If the numerator and the denominator of the fraction are each a perfect square, find the square root of each separately.

Example 1.—Find $\sqrt{\frac{144}{625}}$.

The $\sqrt{144}=12$, and $\sqrt{625}=25$.

Hence,

$$\sqrt{\frac{144}{625}}=\frac{12}{25}.$$

If the numerator and denominator are not each a perfect square, reduce the fraction to a decimal and then extract the square root as in Art. 92.

Example 2.—Find $\sqrt{\frac{2}{7}}$.

Reducing to a decimal, $\frac{2}{7}=0.28571428\ldots$

$$\sqrt{0.28571428}=0.5345$$

Hence,

$$\sqrt{\tfrac{2}{7}}=0.5345 \text{ (to four decimal places)}$$

It is worth noting here that the square root of $\frac{2}{7}$ may be found by extracting the square root of both numerator and denominator, and then dividing the square root of the numerator by the square root of the denominator. This process would require two extractions of roots and one long division and so make the work about three times what it is if the fraction is first reduced to a decimal and then the root extracted.

95. Short methods. *Partly division.*—If it is required to extract the square root of a number to, say, five decimal places, making seven figures in the root, the work may be shortened by extracting the root in the usual

way till four figures are obtained, and then dividing the last remainder found by the corresponding trial divisor to obtain the last three figures of the root. In general, extract the root till more than **half** the required number of figures is found, and then for the other figures of the root divide the remainder by the corresponding trial divisor.

Example 1.—Find $\sqrt{178}$ to five decimal places.

Process

```
      1'78.'00'00'00'00(13.34166
        1
     ------
23   | 78
       69
     ------
263  | 900
       789
     ------
2664 | 11100
       10656
     ------
2668)    444000(166
           2668
         -------
           17720
           16008
         -------
            17120
            16008
         -------
             1112
```

The process may be contracted still further by using contracted division when dividing.

Method by factoring.—When the number of which the square root is to be extracted can be factored into two factors, one of which is a perfect square and the other the number 2, 3, 5, 6, or 7, a very useful short method

may be used. For this purpose it is necessary first to have found the following square roots:

$$\sqrt{2}=1.4142, \qquad \sqrt{3}=1.7320, \qquad \sqrt{5}=2.2360,$$
$$\sqrt{6}=2.4494, \qquad \sqrt{7}=2.6457.$$

Of these the most useful are the roots of 2 and 3.

Example 2.—Find the $\sqrt{32}$.

$32=16\times 2$, so we may write

$$\sqrt{32}=\sqrt{16}\times\sqrt{2}=4\times 1.4142=5.6568$$

Example 3.—Find $\sqrt{125}$.

$$\sqrt{125}=\sqrt{25}\times\sqrt{5}=5\times 2.236=11.180$$

96. Reasons for the methods.—It will be noted that, in the examples of the extraction of square root, the *how* is carefully explained but nothing is said about the *why*. A suggestion as to why the steps are taken as given may be found from the consideration of the square of a number consisting of two figures, tens and units. We choose the number 37 and write it as $30+7$. Find the square of this as follows:

$$\begin{array}{r} 30+7 \\ 30+7 \\ \hline 30\times 7+7^2 \\ 30^2+30\times 7 \\ \hline 30^2+2\times 30\times 7+7^2 \end{array}$$

or

$$30^2+(2\times 30+7)7=900+67\times 7=1369$$

Now extracting the square root of 1369 is the inverse of this process and may be exhibited as follows:

	1369	$30+7=37$, *root.*
$30^2=$	900	
Trial divisor, $2\times30=60$	469	
True divisor, $2\times30+7=67$		
$(2\times30+7)7=$	469	

97. Rule for square root.—After carefully following through the solutions of the preceding examples, the following rule should be understood:

(1) *Begin at the decimal point and point off the whole number part and the decimal part into periods of two figures each. If there is an odd number of figures in the whole number part, the left-hand period will have only one figure. If there is an odd number of figures in the decimal part, annex a* 0 *so that the right-hand period shall contain two figures.*

(2) *Find the greatest square in the left-hand period and place it under that period. The square root of this greatest square is the first figure of the required root. Subtract the greatest square from the left-hand period and bring down and unite with the remainder the next period of the number. This is the first remainder.*

(3) *Take twice the root already found for a trial divisor, which is written at the left of the remainder. Find how many times this trial divisor is contained in the remainder, omitting the right-hand figure. This gives the next figure of the root, which is placed in the root and also at the right of the trial divisor, forming the true divisor. Multiply the true divisor by the figure last placed in the root and write the product under the remainder. Subtract and bring down and unite the next period in the number. This process is repeated for each figure of the root.*

(4) *If at any time the trial divisor will not be contained in the corresponding remainder, place a* 0 *in the root and at the right of the trial divisor; then bring down another period, and continue as before.*

(5) *Point off in the root as many decimal figures as there are decimal periods in the number of which the root is extracted.*

98. Cube root.—The extraction of cube root is so seldom used that it is thought best to omit the usual consideration of it. It is found in a very simple manner by the use of logarithms, by which means any one root is as easily found as another. The extraction of any root by logarithms is found in Part IV.

EXERCISES

Find the square root of each of the following:

1. 622,521. **3.** 0.1444. **5.** 3481.826049.

2. 1,092,025. **4.** 0.127449. **6.** 3, to four decimals.

7. 553,536.2, to two decimals.

8. 990,025.7, to three decimals.

9. 2, to five decimals. **11.** 6, to six decimals.

10. 5, to six decimals. **12.** 7, to six decimals.

13. $\frac{36}{169}$. **14.** $\frac{64}{441}$. **15.** $\frac{25}{625}$. **16.** $\frac{121}{576}$. **17.** $\frac{1681}{9801}$.

18. $27 \div 156.25$, to four decimals.

19. $\frac{7}{9}$, to four decimal places.

In each of Exercises 20 to 27, carry the root to five decimal places:

20. 143. **22.** 92. **24.** 287. **26.** 416.

21. 164. **23.** 278. **25.** 396. **27.** 539.

Find the square roots of the following by short methods:

28. 28.	**30.** 288.	**32.** 147.	**34.** 432.
29. 72.	**31.** 75.	**33.** 192.	**35.** 4107.

36. Find the side of a square rug whose area is 68 sq. ft. Give the result to the nearest $\frac{1}{8}$ in.

37. Find to the nearest $\frac{1}{8}$ in. the side of a square table top containing 13.3 sq. ft.

38. Cool air enters a room through a square pipe which is attached to, and has the same area of cross section as, a rectangular pipe 38 in. by 17 in. What is a side of the square pipe to the nearest $\frac{1}{8}$ in.?

39. Two air pipes have square cross sections 10 in. and 12 in. on a side, respectively. Find the size of a pipe, square in cross section, having the same area as the two pipes.

99. Use of table of square roots.—The use of a table of square roots saves much time and labor. Table IV on page 171 is so formed as to give the square root of any number of not more than four significant figures, correct to four figures; that is, the square root of any number formed from any figure from 1 to 9999, with the decimal point in any position, can be found. It will be found that the last figure of a root when using a number of four figures will occasionally be too large or too small.

The first two figures of a number whose root is to be found are the N column; the third figure, at the top of the page; and the fourth figure, when different from zero, at the top of the right-hand columns. The following will make clear the use of the table:

On page 171, $\sqrt{3.87} = 1.967$.

To find $\sqrt{3.876}$, we add to $\sqrt{3.87}$ the number found in the column headed 6 at the right. In this case the

number is 2 and stands for 0.002. This gives $\sqrt{3.876} = 1.969$.

Since, when a number is multiplied by 100, the square root is multiplied by 10, and when a number is divided by 100, the square root is divided by 10, we have

$\sqrt{387} = 19.67$; $\sqrt{387.6} = 19.69$; $\sqrt{38700} = 196.7$;

$\sqrt{0.0387} = 0.1967$; $\sqrt{0.000387} = 0.01967$.

For any number with the decimal point placed as here, use the table on pages 171 and 172. If the decimal point is placed differently, use pages 173 and 174. Thus, on page 173 we find

$$\sqrt{38.7} = 6.221, \text{ and } \sqrt{38.76} = 6.226.$$

Also $\sqrt{3876} = 62.26$, $\sqrt{387600} = 622.6$, $\sqrt{0.3876} = 0.6226$.

Stated in another way, pages 171 and 172 give *directly* the square roots of numbers from 1 to 9.999, and pages 173 and 174 give *directly* the square roots of numbers from 10 to 99.99. The square roots of other numbers are found by using the principle, that moving the decimal point two places in the number moves the decimal point one place in the square root.

EXERCISES

By using the table of square roots, find the square roots of the following and check by computing the roots in the ordinary manner:

1. 1.41.	**6.** 48.9.	**11.** 0.1234.	**16.** 2900.
2. 1.414.	**7.** 4567.	**12.** 0.03217.	**17.** 13,240.
3. 141.4.	**8.** 12.12.	**13.** 0.0079.	**18.** 67,340.
4. 45.38.	**9.** 1001.	**14.** 0.0202.	**19.** 60,000.
5. 76.35.	**10.** 1113.	**15.** 0.1909.	**20.** 93,080.

100. Similar figures.—The following principle is useful in solving many problems:

The areas of similar figures are in the same ratio as the squares of their like dimensions.

Similar figures are such as have the same shape.

In Fig. 30, the following pairs are similar: (*a*) and (*b*), (*c*) and (*d*), (*e*) and (*f*), (*g*) and (*h*).

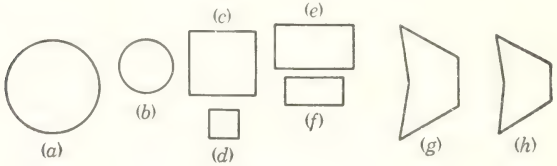

FIG. 30.

The following principle holds for solids:

The volumes of similar solids are in the same ratio as the cubes of their like dimensions.

EXERCISES

1. If the diameter of (*a*) in Fig. 30 is 6 in. and of (*b*) is 4 in., how many times as large as (*b*) is (*a*)?

Solution.—Area of (*a*): area of (*b*) $= 6^2 : 4^2 = 36 : 16 = 2\frac{1}{4}$.

Hence (*a*) is $2\frac{1}{4}$ times as large as (*b*).

2. Find the ratio of the area of (*e*) to the area of (*f*) if the shorter side of (*e*) is 9 ft. and the shorter side of (*f*) 5 ft.

3. A six-cylinder automobile engine with a $3\frac{1}{4}$-in. bore has a computed horsepower of 25.35. What is the horsepower of a six-cylinder engine with a $3\frac{1}{2}$-in. bore?

4. The pipe leading from the carburetor of an automobile is increased in diameter from 1 in. to $1\frac{1}{4}$ in. Find the ratio of the quantity of gas that can be supplied to the engine after the change to that before the change. Disregard the friction in the pipes.

5. Find the ratio of the areas of the cross sections of No. 0 and No. 20 copper wire if their diameters are 0.3125 and 0.0375 in., respectively.

6. If a round steel rod $\frac{1}{2}$ in. in diameter hanging vertically will support 12,000 lb., what will a rod $\frac{7}{8}$ in. in diameter support?

7. Given that the electrical resistance is inversely in the same ratio as the areas of the cross sections of the conductors of the same material; find the ratio of the resistances of two copper wires of diameters $\frac{1}{8}$ in. and $\frac{1}{3}$ in., respectively.

8. Two steam boilers of the same shape are 12 ft. and 18 ft. long, respectively. Find the ratio of their surfaces.

9. How many times as much gold leaf will it take to cover a ball 10 in. in diameter than to cover a ball 6 in. in diameter?

10. Two balls of steel are 7 in. and 15 in. in diameter, respectively. The second is how many times as heavy as the first?

11. Which are the cheaper, oranges $2\frac{1}{2}$ in. through at 30 cents per dozen or 3-in. oranges at 40 cents per dozen? What should the large ones sell at to give the same value for the money as the smaller ones at 30 cents per dozen?

12. Two balls of the same material are 10 in. and 3 in. in diameter, respectively. If the smaller ball weighs 9 lb., what is the weight of the larger?

13. The formula $V = \sqrt{2gh}$ gives the velocity V in feet per second that a body will have after falling from a height h. Find the value of V for a stone that has fallen 400 ft. In the formula $g = 32.2$.

Suggestion.—As in this exercise, the evaluation of a formula often requires the extraction of a square root. The numbers that the letters stand for are put in place of the letters and we have

$$V = \sqrt{2 \times 32.2 \times 400} = \sqrt{25760} = 160.5-.$$

14. The effective area of a chimney is given by the formula

$$E = A - 0.06\sqrt{A}$$

where

E = the effective area.

A = the actual area of the flue.

Find the effective area if $A = 86$ sq. in. If $A = 3.14$ sq. ft.

15. When the pressure of water at the place of discharge is known, the rate of flow is given by the formula

$$V = 12.16\sqrt{P}$$

where

V = velocity of discharge in feet per second.

P = pressure in pounds per square inch at the place of discharge.

Find the rate of discharge if the pressure as given by a pressure gage is 50 lb. per square inch.

16. As in the last exercise, find the velocity of discharge if the pressure is 200 lb. per square inch. Compare the result with that of the preceding.

17. The charge E in volts of a condenser in a radio transmitting station is given by the formula $E = 1000\sqrt{\frac{2W}{C}}$, where W is the energy in joules and C is the capacity of the condenser in microfarads. (*a*) if $W = 0.2$ joule and $C = 0.001$ microfarad, find E. (*b*) If $W = 0.3$ joule and $C = 0.001$ microfarad, find E.

18. The impedance Z in ohms of a radio circuit is given by the formula $Z = \sqrt{R^2 + X^2}$, where R, a resistance, and X, a reactance, are both in ohms. Given $R = 800$ ohms and $X = 1600$ ohms, find Z.

USEFUL TABLES

The following tables are inserted here for reference in solving exercises and for use in practical work. Each will be found self-explanatory.

TABLE I.—DECIMAL AND FRACTIONAL EQUIVALENTS OF PARTS OF AN INCH

8ths	32nds	64ths	
1 = .125	1 = .03125	1 = .015625	33 = .515625
2 = .250	3 = .09375	3 = .046875	35 = .546875
3 = .375	5 = .15625	5 = .078125	37 = .578125
4 = .500	7 = .21875	7 = .109375	39 = .609375
5 = .625	9 = .28125	9 = .140625	41 = .640625
6 = .750	11 = .34375	11 = .171875	43 = .671875
7 = .875	13 = .40625	13 = .203125	45 = .703125
16ths	15 = .46875	15 = .234375	47 = .734375
1 = .0625	17 = .53125	17 = .265625	49 = .765625
3 = .1875	19 = .59375	19 = .296875	51 = .796875
5 = .3125	21 = .65625	21 = .328125	53 = .828125
7 = .4375	23 = .71875	23 = .359375	55 = .859375
9 = .5625	25 = .78125	25 = .390625	57 = .890625
11 = .6875	27 = .84375	27 = .421875	59 = .921875
13 = .8125	29 = .90625	29 = .453125	61 = .953125
15 = .9375	31 = .96875	31 = .484375	63 = .984375

TABLE II.—ENGLISH INCHES INTO MILLIMETERS

In.	0	1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16
0	0.0	1.6	3.2	4.8	6.4	7.9	9.5	11.1	12.7	14.3	15.9	17.5	19.1	20.6	22.2	23.8
1	25.4	27.0	28.6	30.2	31.7	33.3	34.9	36.5	38.1	39.7	41.3	42.9	44.4	46.0	47.6	49.2
2	50.8	52.4	54.0	55.6	57.1	58.7	60.3	61.9	63.5	65.1	66.7	68.3	69.8	71.4	73.0	74.6
3	76.2	77.8	79.4	81.0	82.5	84.1	85.7	87.3	88.9	90.5	92.1	93.7	95.2	96.8	98.4	100.0
4	101.6	103.2	104.8	106.4	108.0	109.5	111.1	112.7	114.3	115.9	117.5	119.1	120.7	122.2	123.8	125.4
5	127.0	128.6	130.2	131.8	133.4	134.9	136.5	138.1	139.7	141.3	142.9	144.5	146.1	147.6	149.2	150.8
6	152.4	154.0	155.6	157.2	158.8	160.3	161.9	163.5	165.1	166.7	168.3	169.9	171.5	173.0	174.6	176.2
7	177.8	179.4	181.0	182.6	184.2	185.7	187.3	188.9	190.5	192.1	193.7	195.3	196.9	198.4	200.0	201.6
8	203.2	204.8	206.4	208.0	209.6	211.1	212.7	214.3	215.9	217.5	219.1	220.7	222.3	223.8	225.4	227.0
9	228.6	230.2	231.8	233.4	235.0	236.5	238.1	239.7	241.3	242.9	244.5	246.1	247.7	249.2	250.8	252.4
10	254.0	255.6	257.2	258.8	260.4	261.9	263.5	265.1	266.7	268.3	269.9	271.5	273.1	274.6	276.2	277.8
11	279.4	281.0	282.6	284.2	285.7	287.3	288.9	290.5	292.1	293.7	295.3	296.9	298.4	300.0	301.6	303.2
12	304.8	306.4	308.0	309.6	311.1	312.7	314.3	315.9	317.5	319.1	320.7	322.3	323.8	325.4	327.0	328.6
13	330.2	331.8	333.4	335.0	336.5	338.1	339.7	341.3	342.9	344.5	346.1	347.7	349.2	350.8	352.4	354.0
14	355.6	357.2	358.8	360.4	361.9	363.5	365.1	366.7	368.3	369.9	371.5	373.1	374.6	376.2	377.8	379.4
15	381.0	382.6	384.2	385.8	387.3	388.9	390.5	392.1	393.7	395.3	396.9	398.5	400.0	401.6	403.2	404.8
16	406.4	408.0	409.6	411.2	412.7	414.3	415.9	417.5	419.1	420.7	422.3	423.9	425.4	427.0	428.6	430.2
17	431.8	433.4	435.0	436.6	438.1	439.7	441.3	442.9	444.5	446.1	447.7	449.3	450.8	452.4	454.0	455.6
18	457.2	458.8	460.4	462.0	463.5	465.1	466.7	468.3	469.9	471.5	473.1	474.7	476.2	477.8	479.4	481.0
19	482.6	484.2	485.8	487.4	488.9	490.5	492.1	493.7	495.3	496.9	498.5	500.1	501.6	503.2	504.8	506.4
20	508.0	509.6	511.2	512.8	514.3	515.9	517.5	519.1	520.7	522.3	523.9	525.5	527.0	528.6	530.2	531.8
21	533.4	535.0	536.6	538.2	539.7	541.3	542.9	544.5	546.1	547.7	549.3	550.9	552.4	554.0	555.6	557.2
22	558.8	560.4	562.0	563.6	565.1	566.7	568.3	569.9	571.5	573.1	574.7	576.3	577.8	579.4	581.0	582.6
23	584.2	585.8	587.4	589.0	590.5	592.1	593.7	595.3	596.9	598.5	600,1	601,7	603.2	604.8	606.4	608.0

39.37 in. = 1 m. = 10 dm. = 100 cm. = 1000 mm.

TABLE III.—SPECIFIC GRAVITIES AND WEIGHTS OF SUBSTANCES

Name of substance	Pounds per cu. in.	Pounds per cu. ft.	Specific gravity
Air	.....	0.0795	
Aluminum	.....	162	2.6
Anthracite coal, broken	.....	52 to 60	
Antimony	.....	418	6.7
Asphaltum	.....	87.3	1.4
Beech wood	.....	46	0.73
Birch wood	.....	41	0.65
Brass, cast (copper and zinc)	.....	506	8.1
Brass, rolled	.....	525	8.4
Brick, common	.....	125	
Brick, pressed	.....	150	
Bronze, copper 8, tin 1 (gun metal)	.....	531	8.5
Chalk	.....	156	2.5
Coal, bituminous, broken	.....	47 to 56	
Coke, loose	.....	23 to 32	
Corundum	.....	.....	3.9
Copper, cast	.....	542	8.6 to 8.8
Copper, rolled	0.319	555	8.8 to 9
Cork	.....	15	0.24
Ebony wood	.....	76	1.23
Elm wood	.....	35	0.56
Flint	.....	162	2.6
Glass	.....	186	2.5 to 3.45
Gold	0.695	.....	19.3
Granite	.....	170	2.56 to 2.88
Hickory wood	.....	53	0.85
Ice	.....	57.5	0.92
Iron, cast	0.26	450	6.9 to 7.4
Iron, wrought	0.28	480	7.69

TABLE III.—SPECIFIC GRAVITIES AND WEIGHTS OF SUBSTANCES.—(*Continued*)

Name of substance	Pounds per cu. in.	Pounds per cu. ft.	Specific gravity
Lead	0.412	712	11.42
Marble	.....	168.7	2.7
Maple wood	.....	49	0.79
Mercury	0.49	.....	13.6
Nickel	0.318	.....	8.8
Oak wood, red	.....	46	0.73 to 0.75
Pine wood, white	....	28	0.45
Pine wood, yellow	...	38	0.61
Platinum	.....	.....	21.5
Quartz	.....	165	2.65
Silver	0.379	655	10.5
Steel	0.29	490	7.85
Tin	.....	459	7.2 to 7.5
Zinc	.....	438	6.8 to 7.2
Water, distilled at 32° F.	.....	62.417	
Water, distilled at 62° F.	.....	62.355	1
Water, distilled at 212° F.	.....	59.7	

SPECIFIC GRAVITIES REFERRED TO AIR

Air	1
Oxygen	1.11
Hydrogen	0.07
Chlorine gas	2.44

SPECIFIC GRAVITIES REFERRED TO HYDROGEN

Hydrogen	1
Air	14.53
Oxygen	15.95
Coal gas	6

TABLE IV.—SQUARE ROOTS

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1.0	1.000	1.005	1.010	1.015	1.020	1.025	1.030	1.034	1.039	1.044	0	1	1	2	2	3	3	4	4
1.1	1.049	1.054	1.058	1.063	1.068	1.072	1.077	1.082	1.086	1.091	0	1	1	2	2	3	3	4	4
1.2	1.095	1.100	1.105	1.109	1.114	1.118	1.122	1.127	1.131	1.136	0	1	1	2	2	3	3	4	4
1.3	1.140	1.145	1.149	1.153	1.158	1.162	1.166	1.170	1.175	1.179	0	1	1	2	2	3	3	3	4
1.4	1.183	1.187	1.192	1.196	1.200	1.204	1.208	1.212	1.217	1.221	0	1	1	2	2	2	3	3	4
1.5	1.225	1.229	1.233	1.237	1.241	1.245	1.249	1.253	1.257	1.261	0	1	1	2	2	2	3	3	4
1.6	1.265	1.269	1.273	1.277	1.281	1.285	1.288	1.292	1.296	1.300	0	1	1	2	2	2	3	3	4
1.7	1.304	1.308	1.311	1.315	1.319	1.323	1.327	1.330	1.334	1.338	0	1	1	2	2	2	3	3	3
1.8	1.342	1.345	1.349	1.353	1.356	1.360	1.364	1.367	1.371	1.375	0	1	1	1	2	2	3	3	3
1.9	1.378	1.382	1.386	1.389	1.393	1.396	1.400	1.404	1.407	1.411	0	1	1	1	2	2	3	3	3
2.0	1.414	1.418	1.421	1.425	1.428	1.432	1.435	1.439	1.442	1.446	0	1	1	1	2	2	2	3	3
2.1	1.449	1.453	1.456	1.459	1.463	1.466	1.470	1.473	1.476	1.480	0	1	1	1	2	2	2	3	3
2.2	1.483	1.487	1.490	1.493	1.497	1.500	1.503	1.507	1.510	1.513	0	1	1	1	2	2	2	3	3
2.3	1.517	1.520	1.523	1.526	1.530	1.533	1.536	1.539	1.543	1.546	0	1	1	1	2	2	2	3	3
2.4	1.549	1.552	1.556	1.559	1.562	1.565	1.568	1.572	1.575	1.578	0	1	1	1	2	2	2	3	3
2.5	1.581	1.584	1.587	1.591	1.594	1.597	1.600	1.603	1.606	1.609	0	1	1	1	2	2	2	3	3
2.6	1.612	1.616	1.619	1.622	1.625	1.628	1.631	1.634	1.637	1.640	0	1	1	1	2	2	2	2	3
2.7	1.643	1.646	1.649	1.652	1.655	1.658	1.661	1.664	1.667	1.670	0	1	1	1	2	2	2	2	3
2.8	1.673	1.676	1.679	1.682	1.685	1.688	1.691	1.694	1.697	1.700	0	1	1	1	1	2	2	2	3
2.9	1.703	1.706	1.709	1.712	1.715	1.718	1.720	1.723	1.726	1.729	0	1	1	1	1	2	2	2	3
3.0	1.732	1.735	1.738	1.741	1.744	1.746	1.749	1.752	1.755	1.758	0	1	1	1	1	2	2	2	3
3.1	1.761	1.764	1.766	1.769	1.772	1.775	1.778	1.780	1.783	1.7	0	1	1	1	1	2	2	2	3
3.2	1.789	1.792	1.794	1.797	1.800	1.803	1.806	1.808	1.811	1.814	0	1	1	1	1	2	2	2	2
3.3	1.817	1.819	1.822	1.825	1.828	1.830	1.833	1.836	1.838	1.841	0	1	1	1	1	2	2	2	2
3.4	1.844	1.847	1.849	1.852	1.855	1.857	1.860	1.863	1.865	1.868	0	1	1	1	1	2	2	2	2
3.5	1.871	1.873	1.876	1.879	1.881	1.884	1.887	1.889	1.892	1.895	0	1	1	1	1	2	2	2	2
3.6	1.897	1.900	1.903	1.905	1.908	1.910	1.913	1.916	1.918	1.921	0	1	1	1	1	2	2	2	2
3.7	1.924	1.926	1.929	1.931	1.934	1.936	1.939	1.942	1.944	1.947	0	1	1	1	1	2	2	2	2
3.8	1.949	1.952	1.954	1.957	1.960	1.962	1.965	1.967	1.970	1.972	0	1	1	1	1	2	2	2	2
3.9	1.975	1.977	1.980	1.982	1.985	1.987	1.990	1.992	1.995	1.997	0	1	1	1	1	2	2	2	2
4.0	2.000	2.002	2.005	2.007	2.010	2.012	2.015	2.017	2.020	2.022	0	0	1	1	1	1	2	2	2
4.1	2.025	2.027	2.030	2.032	2.035	2.037	2.040	2.042	2.045	2.047	0	0	1	1	1	1	2	2	2
4.2	2.049	2.052	2.054	2.057	2.059	2.062	2.064	2.066	2.069	2.071	0	0	1	1	1	1	2	2	2
4.3	2.074	2.076	2.078	2.081	2.083	2.086	2.088	2.090	2.093	2.095	0	0	1	1	1	1	2	2	2
4.4	2.098	2.100	2.102	2.105	2.107	2.110	2.112	2.114	2.117	2.119	0	0	1	1	1	1	2	2	2
4.5	2.121	2.124	2.126	2.128	2.131	2.133	2.135	2.138	2.140	2.142	0	0	1	1	1	1	2	2	2
4.6	2.145	2.147	2.149	2.152	2.154	2.156	2.159	2.161	2.163	2.166	0	0	1	1	1	1	2	2	2
4.7	2.168	2.170	2.173	2.175	2.177	2.179	2.182	2.184	2.186	2.189	0	0	1	1	1	1	2	2	2
4.8	2.191	2.193	2.195	2.198	2.200	2.202	2.205	2.207	2.209	2.211	0	0	1	1	1	1	2	2	2
4.9	2.214	2.216	2.218	2.220	2.223	2.225	2.227	2.229	2.232	2.234	0	0	1	1	1	1	2	2	2
5.0	2.236	2.238	2.241	2.243	2.245	2.247	2.249	2.252	2.254	2.256	0	0	1	1	1	1	2	2	2
5.1	2.258	2.261	2.263	2.265	2.267	2.269	2.272	2.274	2.276	2.278	0	0	1	1	1	1	2	2	2
5.2	2.280	2.283	2.285	2.287	2.289	2.291	2.293	2.296	2.298	2.300	0	0	1	1	1	1	2	2	2
5.3	2.302	2.304	2.307	2.309	2.311	2.313	2.315	2.317	2.319	2.322	0	0	1	1	1	1	2	2	2
5.4	2.324	2.326	2.328	2.330	2.332	2.335	2.337	2.339	2.341	2.343	0	0	1	1	1	1	2	2	2
N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

TABLE IV.—SQUARE ROOTS.—(*Continued*)

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
5.5	2.345	2.347	2.349	2.352	2.354	2.356	2.358	2.360	2.362	2.364	0	0	1	1	1	1	1	2	2
5.6	2.366	2.369	2.371	2.373	2.375	2.377	2.379	2.381	2.383	2.385	0	0	1	1	1	1	1	2	2
5.7	2.387	2.390	2.392	2.394	2.396	2.398	2.400	2.402	2.404	2.406	0	0	1	1	1	1	1	2	2
5.8	2.408	2.410	2.412	2.415	2.417	2.419	2.421	2.423	2.425	2.427	0	0	1	1	1	1	1	2	2
5.9	2.429	2.431	2.433	2.435	2.437	2.439	2.441	2.443	2.445	2.447	0	0	1	1	1	1	1	2	2
6.0	2.449	2.452	2.454	2.456	2.458	2.460	2.462	2.464	2.466	2.468	0	0	1	1	1	1	1	2	2
6.1	2.470	2.472	2.474	2.476	2.478	2.480	2.482	2.484	2.486	2.488	0	0	1	1	1	1	1	2	2
6.2	2.490	2.492	2.494	2.496	2.498	2.500	2.502	2.504	2.506	2.508	0	0	1	1	1	1	1	2	2
6.3	2.510	2.512	2.514	2.516	2.518	2.520	2.522	2.524	2.526	2.528	0	0	1	1	1	1	1	2	2
6.4	2.530	2.532	2.534	2.536	2.538	2.540	2.542	2.544	2.546	2.548	0	0	1	1	1	1	1	2	2
6.5	2.550	2.551	2.553	2.555	2.557	2.559	2.561	2.563	2.565	2.567	0	0	1	1	1	1	1	2	2
6.6	2.569	2.571	2.573	2.575	2.577	2.579	2.581	2.583	2.585	2.587	0	0	1	1	1	1	1	2	2
6.7	2.588	2.590	2.592	2.594	2.596	2.598	2.600	2.602	2.604	2.606	0	0	1	1	1	1	1	2	2
6.8	2.608	2.610	2.612	2.613	2.615	2.617	2.619	2.621	2.623	2.625	0	0	1	1	1	1	1	2	2
6.9	2.627	2.629	2.631	2.632	2.634	2.636	2.638	2.640	2.642	2.644	0	0	1	1	1	1	1	2	2
7.0	2.646	2.648	2.650	2.651	2.653	2.655	2.657	2.659	2.661	2.663	0	0	1	1	1	1	1	2	2
7.1	2.665	2.666	2.668	2.670	2.672	2.674	2.676	2.678	2.680	2.681	0	0	1	1	1	1	1	1	2
7.2	2.683	2.685	2.687	2.689	2.691	2.693	2.694	2.696	2.698	2.700	0	0	1	1	1	1	1	1	2
7.3	2.702	2.704	2.706	2.707	2.709	2.711	2.713	2.715	2.717	2.718	0	0	1	1	1	1	1	1	2
7.4	2.720	2.722	2.724	2.726	2.728	2.729	2.731	2.733	2.735	2.737	0	0	1	1	1	1	1	1	2
7.5	2.739	2.740	2.742	2.744	2.746	2.748	2.750	2.751	2.753	2.755	0	0	1	1	1	1	1	1	2
7.6	2.757	2.759	2.760	2.762	2.764	2.766	2.768	2.769	2.771	2.773	0	0	1	1	1	1	1	1	2
7.7	2.775	2.777	2.778	2.780	2.782	2.784	2.786	2.787	2.789	2.791	0	0	1	1	1	1	1	1	2
7.8	2.793	2.795	2.796	2.798	2.800	2.802	2.804	2.805	2.807	2.809	0	0	1	1	1	1	1	1	2
7.9	2.811	2.812	2.814	2.816	2.818	2.820	2.821	2.823	2.825	2.827	0	0	1	1	1	1	1	1	2
8.0	2.828	2.830	2.832	2.834	2.835	2.837	2.839	2.841	2.843	2.844	0	0	1	1	1	1	1	1	2
8.1	2.846	2.848	2.850	2.851	2.853	2.855	2.857	2.858	2.860	2.862	0	0	1	1	1	1	1	1	2
8.2	2.864	2.865	2.867	2.869	2.871	2.872	2.874	2.876	2.877	2.879	0	0	1	1	1	1	1	1	2
8.3	2.881	2.883	2.884	2.886	2.888	2.890	2.891	2.893	2.895	2.897	0	0	1	1	1	1	1	1	2
8.4	2.898	2.900	2.902	2.903	2.905	2.907	2.909	2.910	2.912	2.914	0	0	1	1	1	1	1	1	2
8.5	2.915	2.917	2.919	2.921	2.922	2.924	2.926	2.927	2.929	2.931	0	0	1	1	1	1	1	1	2
8.6	2.933	2.934	2.936	2.938	2.939	2.941	2.943	2.944	2.946	2.948	0	0	1	1	1	1	1	1	2
8.7	2.950	2.951	2.953	2.955	2.956	2.958	2.960	2.961	2.963	2.965	0	0	1	1	1	1	1	1	2
8.8	2.966	2.968	2.970	2.972	2.973	2.975	2.977	2.978	2.980	2.982	0	0	1	1	1	1	1	1	2
8.9	2.983	2.985	2.987	2.988	2.990	2.992	2.993	2.995	2.997	2.998	0	0	1	1	1	1	1	1	2
9.0	3.000	3.002	3.003	3.005	3.007	3.008	3.010	3.012	3.013	3.015	0	0	0	1	1	1	1	1	1
9.1	3.017	3.018	3.020	3.022	3.023	3.025	3.027	3.028	3.030	3.032	0	0	0	1	1	1	1	1	1
9.2	3.033	3.035	3.036	3.038	3.040	3.041	3.043	3.045	3.046	3.048	0	0	0	1	1	1	1	1	1
9.3	3.050	3.051	3.053	3.055	3.056	3.058	3.059	3.061	3.063	3.064	0	0	0	1	1	1	1	1	1
9.4	3.066	3.068	3.069	3.071	3.072	3.074	3.076	3.077	3.079	3.081	0	0	0	1	1	1	1	1	1
9.5	3.082	3.084	3.085	3.087	3.089	3.090	3.092	3.094	3.095	3.097	0	0	0	1	1	1	1	1	1
9.6	3.098	3.100	3.102	3.103	3.105	3.106	3.108	3.110	3.111	3.113	0	0	0	1	1	1	1	1	1
9.7	3.114	3.116	3.118	3.119	3.121	3.122	3.124	3.126	3.127	3.129	0	0	0	1	1	1	1	1	1
9.8	3.130	3.132	3.134	3.135	3.137	3.138	3.140	3.142	3.143	3.145	0	0	0	1	1	1	1	1	1
9.9	3.146	3.148	3.150	3.151	3.153	3.154	3.156	3.158	3.159	3.161	0	0	0	1	1	1	1	1	1
N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

TABLE IV.—SQUARE ROOTS.—(*Continued*)

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	3.162	3.178	3.194	3.209	3.225	3.240	3.256	3.271	3.286	3.302	2	3	5	6	8	9	11	12	14
11	3.317	3.332	3.347	3.362	3.376	3.391	3.406	3.421	3.435	3.450	1	3	4	6	7	9	10	12	13
12	3.464	3.479	3.493	3.507	3.521	3.536	3.550	3.564	3.578	3.592	1	3	4	6	7	8	10	11	13
13	3.606	3.619	3.633	3.647	3.661	3.674	3.688	3.701	3.715	3.728	1	3	4	5	7	8	10	11	12
14	3.742	3.755	3.768	3.782	3.795	3.808	3.821	3.834	3.847	3.860	1	3	4	5	7	8	9	11	12
15	3.873	3.886	3.899	3.912	3.924	3.937	3.950	3.962	3.975	3.987	1	3	4	5	6	8	9	10	11
16	4.000	4.012	4.025	4.037	4.050	4.062	4.074	4.087	4.099	4.111	1	2	4	5	6	7	9	10	11
17	4.123	4.135	4.147	4.159	4.171	4.183	4.195	4.207	4.219	4.231	1	2	4	5	6	7	8	10	11
18	4.243	4.254	4.266	4.278	4.290	4.301	4.313	4.324	4.336	4.347	1	2	3	5	6	7	8	9	10
19	4.359	4.370	4.382	4.393	4.405	4.416	4.427	4.438	4.450	4.461	1	2	3	5	6	7	8	9	10
20	4.472	4.483	4.494	4.506	4.517	4.528	4.539	4.550	4.561	4.572	1	2	3	4	6	7	8	9	10
21	4.583	4.593	4.604	4.615	4.626	4.637	4.648	4.658	4.669	4.680	1	2	3	4	5	6	8	9	10
22	4.690	4.701	4.712	4.722	4.733	4.743	4.754	4.764	4.775	4.785	1	2	3	4	5	6	7	8	9
23	4.796	4.806	4.817	4.827	4.837	4.848	4.858	4.868	4.879	4.889	1	2	3	4	5	6	7	8	9
24	4.899	4.909	4.919	4.930	4.940	4.950	4.960	4.970	4.980	4.990	1	2	3	4	5	6	7	8	9
25	5.000	5.010	5.020	5.030	5.040	5.050	5.060	5.070	5.079	5.089	1	2	3	4	5	6	7	8	9
26	5.099	5.109	5.119	5.128	5.138	5.148	5.158	5.167	5.177	5.187	1	2	3	4	5	6	7	8	9
27	5.196	5.206	5.215	5.225	5.235	5.244	5.254	5.263	5.273	5.282	1	2	3	4	5	6	7	8	9
28	5.292	5.301	5.310	5.320	5.329	5.339	5.348	5.357	5.367	5.376	1	2	3	4	5	6	7	7	8
29	5.385	5.394	5.404	5.413	5.422	5.431	5.441	5.450	5.459	5.468	1	2	3	4	5	5	6	7	8
30	5.477	5.486	5.495	5.505	5.514	5.523	5.532	5.541	5.550	5.559	1	2	3	4	4	5	6	7	8
31	5.568	5.577	5.586	5.595	5.604	5.612	5.621	5.630	5.639	5.648	1	2	3	3	4	5	6	7	8
32	5.657	5.666	5.675	5.683	5.692	5.701	5.710	5.718	5.727	5.736	1	2	3	3	4	5	6	7	8
33	5.745	5.753	5.762	5.771	5.779	5.788	5.797	5.805	5.814	5.822	1	2	3	3	4	5	6	7	8
34	5.831	5.840	5.848	5.857	5.865	5.874	5.882	5.891	5.899	5.908	1	2	3	3	4	5	6	7	8
35	5.916	5.925	5.933	5.941	5.950	5.958	5.967	5.975	5.983	5.992	1	2	2	3	4	5	6	7	8
36	6.000	6.008	6.017	6.025	6.033	6.042	6.050	6.058	6.066	6.075	1	2	2	3	4	5	6	7	7
37	6.083	6.091	6.099	6.107	6.116	6.124	6.132	6.140	6.148	6.156	1	2	2	3	4	5	6	7	7
38	6.164	6.173	6.181	6.189	6.197	6.205	6.213	6.221	6.229	6.237	1	2	2	3	4	5	5	6	7
39	6.245	6.253	6.261	6.269	6.277	6.285	6.293	6.301	6.309	6.317	1	2	2	3	4	5	6	6	7
40	6.325	6.332	6.340	6.348	6.356	6.364	6.372	6.380	6.387	6.395	1	2	2	3	4	5	6	6	7
41	6.403	6.411	6.419	6.427	6.434	6.442	6.450	6.458	6.465	6.473	1	2	2	3	4	5	5	6	7
42	6.481	6.488	6.496	6.504	6.512	6.519	6.527	6.535	6.542	6.550	1	2	2	3	4	5	5	6	7
43	6.557	6.565	6.573	6.580	6.588	6.595	6.603	6.611	6.618	6.626	1	2	2	3	4	5	5	6	7
44	6.633	6.641	6.648	6.656	6.663	6.671	6.678	6.686	6.693	6.701	1	2	2	3	4	5	5	6	7
45	6.708	6.716	6.723	6.731	6.738	6.745	6.753	6.760	6.768	6.775	1	1	2	3	4	4	5	6	7
46	6.782	6.790	6.797	6.804	6.812	6.819	6.826	6.834	6.841	6.848	1	1	2	3	4	4	5	6	7
47	6.856	6.863	6.870	6.877	6.885	6.892	6.899	6.907	6.914	6.921	1	1	2	3	4	4	5	6	7
48	6.928	6.935	6.943	6.950	6.957	6.964	6.971	6.979	6.986	6.993	1	1	2	3	4	4	5	6	6
49	7.000	7.007	7.014	7.021	7.029	7.036	7.043	7.050	7.057	7.064	1	1	2	3	4	4	5	6	6
50	7.071	7.078	7.085	7.092	7.099	7.106	7.113	7.120	7.127	7.134	1	1	2	3	4	4	5	6	6
51	7.141	7.148	7.155	7.162	7.169	7.176	7.183	7.190	7.197	7.204	1	1	2	3	4	4	5	6	6
52	7.211	7.218	7.225	7.232	7.239	7.246	7.253	7.259	7.266	7.273	1	1	2	3	3	4	5	6	6
53	7.280	7.287	7.294	7.301	7.308	7.314	7.321	7.328	7.335	7.342	1	1	2	3	3	4	5	5	6
54	7.348	7.355	7.362	7.369	7.376	7.382	7.389	7.396	7.403	7.409	1	1	2	3	3	4	5	5	6

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

TABLE IV.—SQUARE ROOTS.—(*Continued*)

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7.416	7.423	7.430	7.436	7.443	7.450	7.457	7.463	7.470	7.477	1	1	2	3	3	4	5	5	6
56	7.483	7.490	7.497	7.503	7.510	7.517	7.523	7.530	7.537	7.543	1	1	2	3	3	4	5	5	6
57	7.550	7.556	7.563	7.570	7.576	7.583	7.589	7.596	7.603	7.609	1	1	2	3	3	4	5	5	6
58	7.616	7.622	7.629	7.635	7.642	7.649	7.655	7.662	7.668	7.675	1	1	2	3	3	4	5	5	6
59	7.681	7.688	7.694	7.701	7.707	7.714	7.720	7.727	7.733	7.740	1	1	2	3	3	4	4	5	6
60	7.746	7.752	7.759	7.765	7.772	7.778	7.785	7.791	7.797	7.804	1	1	2	3	3	4	4	5	6
61	7.810	7.817	7.823	7.829	7.836	7.842	7.849	7.855	7.861	7.868	1	1	2	3	3	4	4	5	6
62	7.874	7.880	7.887	7.893	7.899	7.906	7.912	7.918	7.925	7.931	1	1	2	3	3	4	4	5	6
63	7.937	7.944	7.950	7.956	7.962	7.969	7.975	7.981	7.987	7.994	1	1	2	3	3	4	4	5	6
64	8.000	8.006	8.012	8.019	8.025	8.031	8.037	8.044	8.050	8.056	1	1	2	2	3	4	4	5	6
65	8.062	8.068	8.075	8.081	8.087	8.093	8.099	8.106	8.112	8.118	1	1	2	2	3	4	4	5	5
66	8.124	8.130	8.136	8.142	8.149	8.155	8.161	8.167	8.173	8.179	1	1	2	2	3	4	4	5	5
67	8.185	8.191	8.198	8.204	8.210	8.216	8.222	8.228	8.234	8.240	1	1	2	2	3	4	4	5	5
68	8.246	8.252	8.258	8.264	8.270	8.276	8.283	8.289	8.295	8.301	1	1	2	2	3	4	4	5	5
69	8.307	8.313	8.319	8.325	8.331	8.337	8.343	8.349	8.355	8.361	1	1	2	2	3	4	4	5	5
70	8.367	8.373	8.379	8.385	8.390	8.396	8.402	8.408	8.414	8.420	1	1	2	2	3	4	4	5	5
71	8.426	8.432	8.438	8.444	8.450	8.456	8.462	8.468	8.473	8.479	1	1	2	2	3	4	4	5	5
72	8.485	8.491	8.497	8.503	8.509	8.515	8.521	8.526	8.532	8.538	1	1	2	2	3	3	4	5	5
73	8.544	8.550	8.556	8.562	8.567	8.573	8.579	8.585	8.591	8.597	1	1	2	2	3	3	4	5	5
74	8.602	8.608	8.614	8.620	8.626	8.631	8.637	8.643	8.649	8.654	1	1	2	2	3	3	4	5	5
75	8.660	8.666	8.672	8.678	8.683	8.689	8.695	8.701	8.706	8.712	1	1	2	2	3	3	4	5	5
76	8.718	8.724	8.729	8.735	8.741	8.746	8.752	8.758	8.764	8.769	1	1	2	2	3	3	4	5	5
77	8.775	8.781	8.786	8.792	8.798	8.803	8.809	8.815	8.820	8.826	1	1	2	2	3	3	4	4	5
78	8.832	8.837	8.843	8.849	8.854	8.860	8.866	8.871	8.877	8.883	1	1	2	2	3	3	4	4	5
79	8.888	8.894	8.899	8.905	8.911	8.916	8.922	8.927	8.933	8.939	1	1	2	2	3	3	4	4	5
80	8.944	8.950	8.955	8.961	8.967	8.972	8.978	8.983	8.989	8.994	1	1	2	2	3	3	4	4	5
81	9.000	9.006	9.011	9.017	9.022	9.028	9.033	9.039	9.044	9.050	1	1	2	2	3	3	4	4	5
82	9.055	9.061	9.066	9.072	9.077	9.083	9.088	9.094	9.099	9.105	1	1	2	2	3	3	4	4	5
83	9.110	9.116	9.121	9.127	9.132	9.138	9.143	9.149	9.154	9.160	1	1	2	2	3	3	4	4	5
84	9.165	9.171	9.176	9.182	9.187	9.192	9.198	9.203	9.209	9.214	1	1	2	2	3	3	4	4	5
85	9.220	9.225	9.230	9.236	9.241	9.247	9.252	9.257	9.263	9.268	1	1	2	2	3	3	4	4	5
86	9.274	9.279	9.284	9.290	9.295	9.301	9.306	9.311	9.317	9.322	1	1	2	2	3	3	4	4	5
87	9.327	9.333	9.338	9.343	9.349	9.354	9.359	9.365	9.370	9.375	1	1	2	2	3	3	4	4	5
88	9.381	9.386	9.391	9.397	9.402	9.407	9.413	9.418	9.423	9.429	1	1	2	2	3	3	4	4	5
89	9.434	9.439	9.445	9.450	9.455	9.460	9.466	9.471	9.476	9.482	1	1	2	2	3	3	4	4	5
90	9.487	9.492	9.497	9.503	9.508	9.513	9.518	9.524	9.529	9.534	1	1	2	2	3	3	4	4	5
91	9.539	9.545	9.550	9.555	9.560	9.566	9.571	9.576	9.581	9.586	1	1	2	2	3	3	4	4	5
92	9.592	9.597	9.602	9.607	9.612	9.618	9.623	9.628	9.633	9.638	1	1	2	2	3	3	4	4	5
93	9.644	9.649	9.654	9.659	9.664	9.670	9.675	9.680	9.685	9.690	1	1	2	2	3	3	4	4	5
94	9.695	9.701	9.706	9.711	9.716	9.721	9.726	9.731	9.737	9.742	1	1	2	2	3	3	4	4	5
95	9.747	9.752	9.757	9.762	9.767	9.772	9.778	9.783	9.788	9.793	1	1	2	2	3	3	4	4	5
96	9.798	9.803	9.808	9.813	9.818	9.823	9.829	9.834	9.839	9.844	1	1	2	2	3	3	4	4	5
97	9.849	9.854	9.859	9.864	9.869	9.874	9.879	9.884	9.889	9.894	1	1	2	2	3	3	4	4	5
98	9.899	9.905	9.910	9.915	9.920	9.925	9.930	9.935	9.940	9.945	0	1	1	2	2	3	3	4	4
99	9.950	9.955	9.960	9.965	9.970	9.975	9.980	9.985	9.990	9.995	0	1	1	2	2	3	3	4	4
N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

INDEX

	Page
Accuracy of results	63
Antecedent	131
Approximate results	84
Averages	120
Belts, carrying power	16
Cancellation	10
Checking	89
Consequent	131
Contracted methods	86
Denominator	18
least common	25
Density	145
Discounts	121
Divisors, rules for	4
Electricity, problems in	73
Evolution	151
Exponent	150
Extreme	133
Factor	3
greatest common	4
Feed of cutting tool	54
Fractions	18
decimal	56
improper	19
proper	19
principles of	21
value of	18

	Page
Grade	136
Heights, measuring	138
Horsepower	16
Hydraulic machines	143
Index of root	151
Integer	3
Interest	128
Involution	150
Kilogram	102
Lever	142
Liter	102
Machine screw proportions	75
Mean	133
Measure, English	93
metric	105
Meter	101
Micrometer	78
Mixed number	19
Multiple	4
least common	4
Number, composite	4
even	3
meaning of	2
odd	4
prime	4
Numerator	18
Per cent of error	120
Percentage	114
cases of	115
Power	150
Proportion	133
Pulleys, rules for	14

PAGE
Radical sign 151
Ratio 131
inverse 131
Root, approximate 156
cube 161
square 151
rule for 160

Screw, gearing 52
threads 51
Signs of operation 7
Similar figures 164
Specific gravity 145
tables of 169
Square, perfect 152
Steam boilers 141
Stevin, Simon 57

Tables of square roots 171
use of 162

Vernier 80
Voids 125

Water pressure 98
Weights, of round iron 48
tables of 94
Worm wheel 53

ANSWERS

Pages 6, 7. Art. 5.

6. 2, 4; 4. **7.** 2, 4, 8; 8. **8.** 3, 9; 9. **9.** 2, 5, 10, 11, 110; 110. **10.** 2, 3, 4, 6, 8, 12, 24; 24. **11.** 5; 5. **12.** 120. **13.** 315. **14.** 2178. **15.** 441. **16.** 225. **17.** 315,084. **18.** 2, 3, 7, etc.

Pages 8–10. Art. 6.

1. 29. **2.** 21. **3.** 0. **4.** 13. **5.** 1. **6.** 162. **7.** 16. **8.** 363. **15.** $73,850. **16.** $35. **17.** 11,037,600. **18.** 1476 revolutions per minute. **19.** 68,640 revolutions. **20.** 1,500,000 pounds per square inch.

Pages 12–14. Art. 7.

1. 6. **2.** 24. **3.** 44. **4.** $82\frac{26}{27}$. **5.** 192. **6.** $2\frac{1}{4}$. **7.** $2\frac{37}{42}$. **8.** $3214\frac{2}{7}$. **9.** $77\frac{8}{11}$. **10.** $1319\frac{1}{9}$. **11.** $8\frac{5}{21}$. **12.** $116\frac{23}{24}$. **13.** 1. **14.** 25. **15.** 1. **16.** $\frac{11}{1152}$. **18.** $117. **19.** 18 pounds. **20.** 132. **21.** 64 boxes. **22.** 128 quarts. **23.** 96 miles. **24.** 48 barrels. **25.** 24 men. **26.** 43 books. **27.** 45 dozen. **28.** 10,080 pounds.

Pages 15–17. Art. 8.

1. 60 revolutions per minute. **2.** 128 revolutions per minute. **3.** 54 revolutions per minute. **4.** 392. **5.** 360. **7.** $10\frac{1}{2}$ inches. **8.** $12\frac{3}{8}$ inches. **9.** 64. **10.** $387\frac{25}{33}$. **12.** 72 horsepower. **13.** $96\frac{9}{10}$ horsepower.

Page 23. Art. 18.

5. $\frac{112}{121}$. **6.** $\frac{8}{9}$. **7.** $\frac{3}{4}$. **8.** $\frac{2}{3}$. **9.** $\frac{1}{14}$. **10.** $\frac{1}{4}$. **11.** $\frac{1}{4}$. **12.** $\frac{1}{2}$. **13.** $\frac{7}{10}$. **14.** $\frac{202}{707}$. **15.** $\frac{117}{77}$. **16.** $\frac{1042}{1097}$. **17.** $\frac{29}{310}$. **18.** $\frac{45}{121}$. **19.** $\frac{8}{9}$. **20.** $\frac{5901}{2122}$.

Page 26. Art. 20.

1. $\frac{6}{12}$; $\frac{4}{12}$; $\frac{2}{12}$. **4.** $\frac{18}{24}$; $\frac{20}{24}$; $\frac{21}{24}$. **7.** $\frac{60}{90}$; $\frac{54}{90}$; $\frac{75}{90}$; $\frac{42}{90}$.
11. $\frac{7}{14}$; $\frac{6}{14}$. **12.** $\frac{25}{30}$; $\frac{28}{30}$. **13.** $\frac{111}{117}$; $\frac{143}{117}$. **14.** $\frac{65}{91}$; $\frac{77}{91}$.
15. $\frac{102}{187}$; $\frac{143}{187}$. **16.** $\frac{209}{323}$; $\frac{221}{323}$. **17.** $\frac{1648}{2472}$; $\frac{1854}{2472}$; $\frac{2060}{2472}$; $\frac{231}{2472}$.
18. $\frac{594}{1620}$; $\frac{351}{1620}$; $\frac{80}{1620}$; $\frac{3150}{1620}$. **19.** $3\frac{75}{120}$; $6\frac{10}{120}$; $7\frac{27}{120}$.

Page 28. Art. 21.

1. $3\frac{5}{7}$. **2.** $5\frac{4}{9}$. **3.** $3\frac{5}{7}$. **4.** $2\frac{1}{3}$. **5.** $15\frac{11}{12}$. **6.** $25\frac{5}{6}$. **7.** $49\frac{1}{4}$.
8. $28\frac{11}{12}$. **9.** $3\frac{2}{5}$. **10.** $4\frac{23}{28}$. **11.** $257\frac{1}{6}$. **12.** $15\frac{15}{28}$.
13. $593\frac{27}{32}$. **14.** $166\frac{1}{6}$. **15.** $5\frac{1}{8}$. **16.** $42\frac{1}{4}$. **17.** $988\frac{2}{6}\frac{2}{5}$.
18. $474\frac{31}{81}$. **19.** $269\frac{3}{5}$. **20.** $180\frac{137}{270}$. **21.** $84\frac{1}{21}$.
22. $484\frac{9}{40}$ yards. **23.** $916\frac{11}{16}$ miles. **24.** $43\frac{19}{24}$ yards.
25. $24\frac{15}{16}$ inches.

Pages 30–32. Art. 22.

1. $\frac{1}{3}$. **2.** $\frac{1}{2}$. **3.** $\frac{1}{8}$. **4.** $\frac{3}{10}$. **5.** $\frac{3}{16}$. **6.** $\frac{2}{3}$. **7.** $3\frac{7}{10}$. **8.** $\frac{1}{12}$.
9. $1\frac{9}{32}$. **10.** $1\frac{71}{90}$. **11.** $\frac{99}{2189}$. **12.** $2\frac{47}{72}$. **13.** $4\frac{1}{30}$. **14.** $65\frac{71}{72}$.
15. $3\frac{91}{100}$. **16.** $296\frac{7}{8}$. **17.** $5999\frac{2}{3}$. **18.** $90\frac{5}{24}$. **19.** $119\frac{1}{12}$.
20. $88\frac{7}{34}$. **21.** $153\frac{23}{24}$. **22.** $8\frac{37}{40}$. **23.** $2000\frac{47}{78}$. **24.** $110\frac{11}{15}$.
25. $30\frac{1}{28}$. **26.** $2\frac{71}{140}$. **27.** $9\frac{4}{5}$. **28.** $23\frac{23}{26}$. **29.** $8\frac{3}{5}$. **30.** $6\frac{3}{8}$.
31. $65\frac{27}{288}$. **32.** $13\frac{1}{9}$. **33.** $21\frac{1}{2}$. **34.** $21\frac{107}{168}$. **35.** $52\frac{1}{2}$.
47. $3\frac{11}{24}$.

Pages 36–39. Art. 25.

31. \$10,120. **32.** 150 gallons. **33.** $\$12\frac{1}{5}$. **34.** 44 feet; 88 feet; $\frac{1}{7}$ foot. **35.** 34 inches; $25\frac{1}{2}$ inches; 1190 feet. **36.** \$150 camping; \$5 hat; \$95 traveling. **37.** \$2.40, skates; \$2.70, coat; 80 cents, book; \$1.30 left. **38.** $2\frac{9}{13}$. **39.** $9\frac{9}{20}$. **40.** 2000. **41.** 586.
42. 37,130. **43.** 8541. **44.** $141\frac{9}{25}$. **45.** $195\frac{1}{15}$. **46.** $1001\frac{5}{12}$.
47. $1755\frac{5}{21}$. **48.** $312\frac{1}{2}$. **49.** $52{,}032\frac{7}{10}$. **52.** $\frac{3}{10}$. **53.** $\frac{3}{4}$. **54.** $\frac{2}{5}$.
55. $41\frac{47}{48}$. **56.** $508\frac{2}{5}$ cents. **57.** 94,500 revolutions; 10,512,000 revolutions. **58.** Hire the boy, $1\frac{5}{8}$ cents cheaper. **59.** \$1679.04.
60. $1134\frac{7}{12}$ miles. **61.** $304\frac{37}{150}$ cubic yards. **62.** $286\frac{10}{29}$; $47\frac{21}{29}$, $11\frac{27}{29}$. **63.** $46\frac{1}{4}$ pounds; 1110 pounds; $93\frac{3}{4}$ pounds.

64. $\$88\frac{11}{36}$. **65.** $\$5\frac{11}{32}$ spent; $\$1\frac{11}{160}$ lost; $\$2\frac{11}{20}$ brought home. **66.** $1086\frac{6}{10}\frac{8}{10}\frac{7}{10}$ feet per second; $4709\frac{1}{7}\frac{9}{0}$ feet per second. **67.** $\$10\frac{1}{32}$, labor cost; $\$3\frac{3}{32}\frac{9}{0}$, materials cost; $\$3\frac{3}{32}\frac{9}{0}$, incidentals. **68.** $2\frac{1}{16}$ miles. **69.** $49.764. **70.** (*a*) $872; (*b*) 66 cents.

Pages 42–50. Art. 28.

41. $\frac{2}{35}\frac{7}{5}$; $2\frac{1}{12}$. **42.** $2\frac{8}{11}$ hours. **43.** $3\frac{1}{5}$ hours. **44.** $3\frac{1}{5}$ hours. **45.** $\frac{2}{12}\frac{5}{6}$. **46.** $215\frac{5}{11}$. **47.** 294. **48.** $23\frac{1}{21}\frac{7}{}$. **49.** $\frac{1}{3}\frac{2}{7}$. **50.** $12\frac{1}{50}$. **51.** $3\frac{1}{2}\frac{2}{7}\frac{7}{5}$. **52.** $\frac{8}{13}\frac{3}{9}$. **53.** $1\frac{1}{90}$. **54.** 4211. **55.** $2\frac{1}{50}$. **56.** $3\frac{1}{22}$. **57.** 3. **58.** $70\frac{11}{136}$. **60.** $\frac{2}{11}\frac{4}{}$. **61.** $\frac{2}{10}\frac{0}{2}\frac{4}{9}\frac{8}{5}\frac{5}{}$. **62.** $55\frac{5}{18}\frac{7}{7}\frac{9}{5}\frac{7}{8}$. **63.** 102,960. **64.** $\frac{11}{12}$. **65.** $95\frac{11}{20}$. **66.** $\frac{2}{3}$; 1; $\frac{1}{4}$; 5. **67.** $\frac{19}{36}$. **68.** $2\frac{26}{33}$. **69.** $2\frac{39}{371}$. **70.** $\frac{6}{26}\frac{8}{1}$. **72.** $15\frac{3}{11}$. **73.** 283. **74.** $\frac{7}{15}$. **75.** $3\frac{1}{5}$. **76.** $9\frac{52}{56}$. **77.** $\frac{4}{11}$. **78.** $\frac{16}{42}\frac{7}{6}\frac{0}{2}\frac{9}{4}$. **79.** $2\frac{1}{6}$. **80.** $1\frac{25}{71}$. **81.** $\frac{7}{11}\frac{9}{9}\frac{3}{2}\frac{}{3}\frac{}{2}$. **82.** $\frac{5}{19}\frac{0}{19}\frac{11}{}$. **83.** $\frac{5}{58}\frac{7}{7}\frac{7}{8}\frac{5}{5}\frac{0}{4}\frac{1}{}$. **84.** $\frac{2}{34}\frac{8}{6}\frac{5}{0}\frac{4}{0}\frac{3}{}$. **85.** $\frac{7}{61}$. **86.** $\frac{3}{16}$, materials; $\frac{45}{64}$, labor; $\frac{1}{16}$, advertising; $\frac{3}{64}$, transportation. **87.** $867\frac{54}{111}$. **88.** 27. **89.** $10\frac{23}{56}$ inches. **90.** $735\frac{65}{123}$ miles. **91.** Use one 10 feet and one 16 feet; or one 12 feet and one 14 feet. **92.** 15 minutes. **97.** $8\frac{4}{7}$ hours. **98.** $1\frac{5}{9}$ hours. **99.** $2\frac{2}{11}$. **100.** $4\frac{1}{3}$ hours both working together. Both can do $\frac{3}{4}$ in 1 hour. **101.** Both working can buy the car in $35\frac{5}{9}\frac{5}{4}\frac{7}{}$ days. **102.** Land, $\$\frac{2}{2}\frac{4}{10}\frac{1}{9}$; building, $\$\frac{5}{5}\frac{5}{2}\frac{0}{3}$. **103.** 48. **104.** 20. **105.** $833.51\frac{5}{8}$. **106.** $10\frac{1}{2}\frac{2}{}$ pounds; $42\frac{1}{8}\frac{4}{}$ pounds; $183\frac{1}{12}$ pounds. **108.** 212 degrees Fahrenheit; 1743 degrees Fahrenheit. **109.** $27\frac{1}{2}$ degrees, 960 degrees. **110.** $\frac{3}{5}$ inch. **111.** $\frac{5}{8}$ inch, approximately. **112.** 50 inches.

Pages 53–55. Art. 31.

4. $\frac{1}{360}$ inch; $\frac{1}{30}$ inch; $\frac{1}{24}$ inch; $19\frac{1}{5}$ turns. **5.** $\frac{1}{16}$ inch; $\frac{1}{16}$ inch; 28 turns. **6.** $\frac{8}{21}$ inch; $1\frac{1}{7}$ inches. **7.** $28\frac{4}{11}$. **9.** $22\frac{2}{9}$. **10.** $5\frac{1}{32}$. **11.** 1. **12.** $28\frac{4}{17}$ minutes. **13.** $6\frac{3}{4}$. **14.** $\frac{5}{32}$ inch. **15.** $19\frac{1}{16}$ minutes. **16.** $\frac{3}{16}$ minute. **17.** $3\frac{3}{10}$ minutes. **18.** $4\frac{3}{8}$ minutes. **19.** 50 feet a minute. **20.** $\frac{15}{26}$ minute. **21.** $28\frac{2}{17}$ minutes. **22.** $\frac{15}{11}$ inch. **23.** 192. **24.** $23\frac{5}{13}$. **25.** $\frac{1}{70}$ inch.

Page 61. Art. 36.

13. $\frac{5}{8}$, $\frac{7}{8}$, $\frac{47}{64}$. **14.** $6\frac{2}{3}$, $\frac{201}{20000}$, $\frac{7}{8}$, $\frac{133}{16000}$. **15.** $\frac{5}{8}$, $1\frac{2}{3}$, $\frac{1}{8}$, $\frac{3}{400}$. **17.** $\frac{10}{64}$, $\frac{7}{64}$, $15\frac{40}{64}$, $1\frac{24}{64}$, $\frac{5}{64}$.

Pages 64–75. Art. 41.

4. 162.425. **5.** 590. **6.** 563.510. **7.** 0.30103. **8.** 0.69897. **9.** 0.015588. **10.** 1.7274. **11.** 0.28980. **12.** 20.6375. **13.** 99.993. **14.** 8,999,108.991. **15.** 99.09. **16.** 9.450625. **17.** 0.013394. **18.** 0.0160655. **19.** 0.00000114. **20.** 38.353+. **21.** 1786.564. **22.** 8.803+. **24.** 11.73. **25.** 0.5313. **26.** 785.02242. **27.** 0.0887−. **28.** 0.9886+. **29.** 38.1157−. **30.** 0.00033+. **31.** 234.7899. **33.** 25. **34.** 199.6927−. **36.** 1.9695+. **37.** 27.3871−. **38.** 7.9257+. **39.** 13.8305 **40.** 0.4710+. **45.** 3000. **46.** 2764.608. **47.** 0.0216. **48.** 1795.20. **49.** 55.25+. **50.** $4.179. **51.** $897,345. **52.** $58.50. **53.** $838.50. **54.** 6.465. in. **55.** 8.889+ mi. **56.** 16,202.016 pounds. **57.** 23.241− pounds of copper; 1.751− pounds of tin; 0.125+ pound of phosphorus. **58.** 9.10625. **59.** 4.756+. **60.** 0.0005128+. **61.** 6.405+. **62.** 424,573.5−. **64.** 387.654321. **65.** 416.666+ hours. **66.** 150+. **67.** $67,719.17. **68.** 8.56− pounds. **69.** $3224.45. **70.** 10,224,000; 1022.4. **71.** 88,593+ pounds. **72.** Tin, 76; Cu. 23; Zn. 833; Ant. 38; Pb. 30. **73.** $18,000, friends; $10,000, cookies; $6,000, shops; $6,000, community welfare. **74.** 0.324 inch. **75.** $192 paid; $64 to be paid. **76.** 85; 5000.94. **77.** $52.42. **78.** 0.016 cubic foot; 0.368 cubic foot. **79.** 0.0171 cubic foot; 0.857 cubic foot. **80.** 1.087−; 0.92. **81.** $1500 each earns: $450 rent; $420 food; $180 travel; $120 cloth. **83.** $\frac{11}{32}$; $\frac{17}{64}$; $\frac{15}{64}$; $\frac{3}{16}$; $\frac{3}{32}$; $\frac{1}{64}$. **84.** $42,500. **85.** $3741.75. **86.** (*a*) 6.168 inches; (*b*) 93; (*c*) 3034.125 pounds. **87.** 1.0024 inches; 670; 2173.5 pounds. **88.** 5046 feet. **89.** 5303 feet. **90.** $139.13. **91.** 0.192 inch. **92.** 2.1823 inches. **93.** 81,000 pounds; 0.0875 inch. **94.** $230.40. **95.** $144. **96.** 25; 16; 14; 11; 8.

97. 0.0095 − inch. **98.** $32\frac{2}{3}$. **99.** 7.35 − square inches.
100. 0.845 inch; 0.4875 inch; 0.1625 inch. **101.** $\frac{5}{8}$.
102. 7.045 − . **103.** 0.0066 + . **105.** 28.05. **106.** 14.4 cents.
107. $2.08. **108.** $3.93. **109.** 190,080 calories. **110.** 99,000 calories; $\frac{11}{32}$ cent. **111.** 2.553 + ohms. **112.** 19.35 + ohms.
113. 116.13 − ohms.

Pages 82, 83. Art. 45.

1. 0.192 inch. **2.** 0.139 inch. **3.** 0.115 inch. **6.** 0.1806 inch.
7. 0.3423 inch. **8.** 0.4157 inch. **9.** 0.2438 inch.

Page 86. Art. 46.

1. 9.8696. **2.** 2518. **3.** 31.83. **4.** 16,034. **5.** 1212.4.
6. 0.3412. **7.** 0.014955.

Pages 95–98. Art. 51.

1. 1007 in. **2.** 66,035 sec. **3.** 139,769 oz. **4.** 575 pt.
5. $59{,}144\frac{3}{4}$ sq. yd. **6.** 7 sq. yd. 4 sq. ft. 84 sq. in. **7.** 134 cu. yd. 3 cu. ft. 1240 cu. in. **8.** 4576. **9.** $20,280. **10.** 2.375.
11. 9490 +. **12.** 8.355 + lb. **13.** 24. **14.** 115.035 −.
15. 0.0534 in. **16.** 0.0346 + . **17.** 1 hr. 8 min. 11 sec. **18.** 144.
19. 0.002604 + ; 0.0104 + ; 0.00087 − ; 0.00347 − . **20.** 0.0833 + ; 0.1667 − ; 0.2917 − ; 0.61458 − . **22.** 0.046875; 0.09375; 0.1875; 0.46875; 0.8125; 0.28125. **23.** 0.2 − . **24.** 33,408. **25.** 46 min. past 11 A.M. **26.** 44. **27.** 1.056 sec. **28.** 0.00262 + .
29. 9.59 − min. **30.** $4.50. **31.** 665.4. **32.** 176.9406 + .
33. 19 miles approx. **34.** 63. **36.** $32.43. **37.** 900.
38. 127 bbl. $45\frac{1}{2}$ lb. **39.** 1.211; 1.24446. **40.** $34\frac{2}{7}$. **41.** 1.3694.
42. $15\frac{19}{41}$. **43.** 612,480; 336,864 tons. **44.** 184,163 miles.
45. 2116.8 lb. **46.** 2.3 ft. approx. **47.** 25 sq. in. **48.** 138.9 − lb.
49. 320,000 ft. lb. **50.** 3,158,250.

Page 99. Art. 52.

1. 19.51 lb.; 8824.7 lb. **2.** 28,123 lb. **3.** 106,493 lb.; 25,123 lb.
4. 375 lb.; no. **5.** 576 ft.

Pages 111–113. Art. 65.

1. 4.56 m., 17,630 m., 27,000 m.; 4560 mm., 17,630,000 mm., 27,000,000 mm. **2.** 0.0075 $m.^2$, 0.000125 $m.^2$, 2.5 $m.^2$ **3.** Approximately. **4.** 416.7 m. **5.** 1.756 $m.^3$, 467 $m.^3$, 0.4937 $m.^3$, 10.67 $m.^3$, 735.432 $m.^3$, 764,000 $m.^3$ **6.** 792.51 qt.; 296.034 qt.; 2952.75 in.; 18.8976 ft.; 766.19 bu.; 0.00245 gal.; 100,636.19 bbl. **7.** 11.58 m.; 1325.88 cm.; 1237.84 l.; 102.27 l.; 26.64 $m.^3$ **9.** 0.025 in., 16 oz. **10.** 0.0005 in. **11.** 2.953− in. **12.** 2 min. 30.2 sec. **13.** 355.58+. **14.** 444.8 kg. **15.** 1.0333− kg. **16.** 4.74. **17.** 74.2 min. **18.** 2.83+ $m.^3$ **19.** 36,000 kg. **20.** 67,500 lb. **22.** 1440 l. **23.** 14.83− mm. **24.** 73.551. **25.** 5 ft. 8.5+ in. **26.** 75.946 cm., 76.200 cm., 76.454 cm., 76.708 cm. **27.** 28.00 in., 29.00 in., 29.50 in. **28.** 44 g. **29.** 981.5−. **30.** 0.7664−, 12.77+. **31.** 1.243−. **32.** 1033.5+.

Pages 117, 118. Art. 69.

24. 10. **25.** 20. **26.** 30. **27.** 50. **28.** $33\frac{1}{3}$. **29.** $33\frac{1}{3}$. **30.** $14\frac{2}{7}$. **31.** 300. **32.** $\frac{2}{3}$; $66\frac{2}{3}\%$. **33.** $\frac{1}{3}$; $33\frac{1}{3}\%$. **34.** 25%. **36.** $87\frac{1}{2}\%$. **37.** 4−%. **38.** $2. **39.** 7−%. **40.** $33\frac{1}{3}\%$. **41.** $3900.

Pages 121–128. Art. 72.

1. 960. **2.** 15.25 cents. **3.** $15.25. **4.** 1200. **5.** $17\frac{6}{7}$. **6.** 100. **7.** $89.60. **8.** 32,800. **9.** 3.36+. **10.** 42.5+. **11.** $87.14. **12.** $1\frac{1}{3}$; $2\frac{2}{4\,9}$. **13.** 6. **14.** 5% approx. **15.** 0.0308 in. **16.** $18\frac{2}{1\,1}$; $1\frac{4\,9}{5\,1}$. **17.** 0.85+. **18.** $71.50. **19.** 0.245, etc. **20.** 9−. **21.** 3.82\frac{1}{2}$; $1.80; $5.60. **22.** 52.8−. **23.** 224−. **24.** 1.04+. **25.** 1647. **26.** $11,450. **27.** 10.5; 0.5. **28.** 3000.8825 lb. of iron; 240.0575 lb. of nickel; 34.06 lb. of other materials. **30.** 10.6+. **31.** $373\frac{1}{3}$. **32.** $69\frac{4}{9}$ billion dollars; 28%. **33.** $26.28; 60.4. **34.** $800; $24. **35.** $665. **36.** $103\frac{1}{3}$ lost. **38.** 1.43; 1.54; 1.67; 1.82; 2.22. **39.** 47−; 17−. **40.** $3.80.

41. $1.3552. **42.** $7000. **43.** $6\frac{1}{16}$%. **44.** $131.40; $197.59
45. $54; $45. **46.** $18,533.76; $231.67. **47.** 31%, or better 30%.
48. Copper $3\frac{7}{11}$%, 86.4 lb.; antimony $8\frac{2}{11}$%, 194.4 lb.; tin $88\frac{2}{11}$%, 2095.2 lb. **49.** 110.9−. **50.** 15.43+%. **51.** 0.02959+%; 0.1057%. **52.** $2.62. **53.** 1564. **54.** 57.44+%, 42.56−%.
55. 0.8078 g., 2.4592 g. **56.** $249.75. **57.** 0.047 ; 29.99 ft.
58. 549.024 lb.; 3.584 lb. **60.** $48\frac{3}{4}$%; 5%; 10.26%. **62.** 68.7 .
63. 231 ft. approx. **64.** 2491 ft. approx. **65.** 0.61%.

Pages 129, 130. Art. 73.

1. $84.00. **2.** $0.7865. **3.** $25.95. **4.** $26.39. **5.** $29.42.
6. $9.955. **7.** $450. **8.** $10.20. **9.** $230.0625. **10.** $437.775.

Pages 132, 133. Art. 74.

6. 50, 30. **7.** 47, 192. **8.** 6.5−%; 11 to 17. **9.** $50, man; $10, son. **11.** $88\frac{1}{3}$%. **12.** 89.18 hp. **13.** 745 watts approx.
14. 91.2+%. **15.** 6.5 and 7.5 amperes.

Pages 137, 138. Art. 76.

1. 9. **2.** 36. **3.** 0.625. **4.** 4.8. **5.** $73\frac{2}{3}$. **6.** 0.075.
7. 50 miles. **8.** 8 days. **9.** 147.2 miles; 83.2 min.
10. $1\frac{1}{9}$% grade; 90. **11.** 85. **12.** $31\frac{1}{2}$ gal. **13.** $42\frac{2}{3}$, 32, $21\frac{1}{3}$.
14. $2400. **15.** $166\frac{2}{3}$. **16.** $13.02. **17.** $12\frac{1}{2}$. **18.** $391\frac{2}{3}$.
19. $74\frac{1}{2}$. **20.** 0.3175 in. **22.** 45 . **23.** 1,180.95 . **24.** 872.

Pages 140, 141. Art. 77.

2. 119 ft. approx. **3.** 60 ft. **4.** 131 ft. approx. **5.** 192 ft.
6. $583\frac{1}{2}$ ft.

Pages 141, 142. Art. 78.

1. 34.5; 2:69. **2.** 103.5. **3.** 11.5. **4.** 8625. **5.** $16\frac{2}{3}$ sq. ft.; 575 sq. ft.

Pages 142, 143. Art. 79.

1. 1250 lb. **2.** 2 ft. 6.2− in. **3.** 7 ft. $\frac{3}{8}$ in. **4.** 381 lb. approx.
5. 560 lb. **6.** $333\frac{1}{3}$ lb. **7.** 2 lb. 9.2− oz. **8.** $62\frac{1}{4}$ lb. **9.** 130 lb.

Page 144. Art. 80.

1. $818\frac{2}{3}$ lb. **2.** 83,160 lb. **3.** 23,091 lb.

Pages 148, 149. Art. 84.

1. 3.828 lb.; 9450 lb.; 73.458 lb. **2.** $2\frac{2}{3}$. **3.** 2.773 lb. **4.** 5.
5. $36\frac{1}{24}$ lb. **6.** 330.75 lb. **7.** 1.26. **8.** 0.409. **10.** 4.424+.
11. 89.75%. **12.** 1.8. **13.** 1000 cc.; 61+ cu. in.
14. 750; 1380 kg. **15.** \$1.56. **16.** 1033.3 g. **17.** 29.89 in.
18. 2175.95 kg. **19.** 2742 lb. **20.** 2835 lb. **21.** 30 lb.
22. 111 lb. approx. **23.** $13\frac{3}{4}$ lb.

Pages 161, 162. Art. 98.

1. 789. **2.** 1045. **3.** 0.38. **4.** 0.357. **5.** 59.007. **6.** 1.7320.
7. 744.00. **8.** 995.000. **9.** 1.414217. **10.** 2.236068.
11. 2.449489. **12.** 2.645751. **13.** $\frac{6}{13}$. **14.** $\frac{8}{21}$. **15.** $\frac{1}{5}$.
16. $\frac{11}{24}$. **17.** $\frac{41}{99}$. **18.** 0.4157. **19.** 0.8819. **20.** 11.95826.
21. 12.80624. **22.** 9.59166. **23.** 16.67333. **24.** 16.94107.
25. 19.89975. **26.** 20.39608. **27.** 23.21637. **28.** 5.2915.
29. 8.485. **30.** 16.971. **31.** 8.6603. **32.** 12.1244. **33.** 13.8564.
34. 20.7846. **35.** 64.086. **36.** 8 ft. 3 in. **37.** 3 ft. $7\frac{7}{8}$ in.
38. $25\frac{3}{8}$ in. **39.** $15\frac{5}{8}$ in.

Pages 164–166. Art. 100.

2. 3.24. **3.** 29.40. **4.** $1\frac{9}{16}$. **5.** $69\frac{4}{9}$. **6.** 36,750 lb. **7.** 64:9.
8. 4:9. **9.** $2\frac{7}{9}$. **10.** 9.84. **11.** 3-in. oranges; 52 cents.
12. $333\frac{1}{3}$ lb. **14.** 85.44 sq. in.; 3.03 sq. ft. **15.** 85.98 ft. per second. **16.** 171.97 ft. per second. **17.** (*a*) 20,000 volts; (*b*) 24,495− volts. **18.** 1789− ohms.

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